2 - Groups in 5 & 6d

Based on: *2105.08724 & 2110.14647 w. L. Bhardwaj, D. Gould, J. Oh, S. Schäfer-Nameki*

- Standard Global Symmetries constrain the spectrum, Local Operators. ($\mathcal{F}$ Flavor)

- 1-form symmetries constrain the Line defects of the theory ($\Gamma^{(1)}$)

- 2-groups: $\mathcal{F} \times_{2\text{-group}} \Gamma^{(1)}$ ($\Gamma^{(1)}$ **DISCRETE** or continuous)

  (Continuous Flavor Symmetries and 1-form symmetry mix with each other)

Many examples in 3 & 4 Dimensions. [Bernini, Cordova, Hsin; Lam, Hsin; Lee, Ohmori, Tachikawa]

Useful in d>4, strongly coupled field theories (Superconformal: SCFTs, Little String: LST) no Lagrangian

Description, only effective at low-energy (Gauge Theories). 2-Group involves non-pert States

Outline

1. 2-groups, definition and construction

2. 5d SU(2) theory

3. 5d 2-Groups, perturbative vs Non-Perturbative

4. 6d Theories with 2-Groups and Classification

1. 2-groups, definition and construction

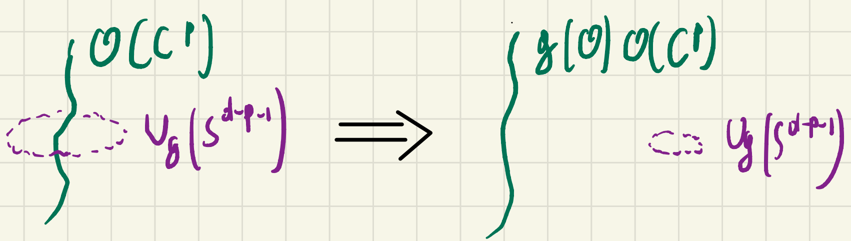
p-Form Symmetries: $\mathcal{O}(C^p)$ (Charged p-dim. operator) U_g (Topological Symmetry operator)

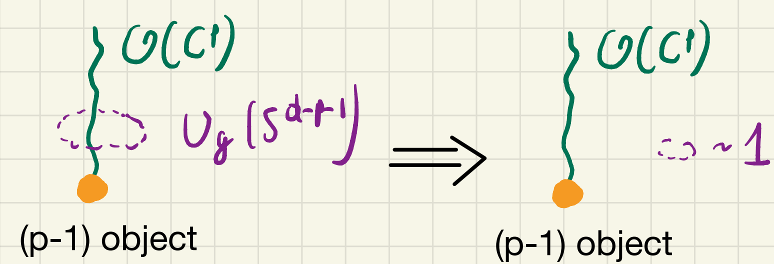
$$[U_g, T_{\mu\nu}]=0$$

- Group law: $U_g(M^{d-p-1})\,U_{g'}(M^{d-p-1}) = U_{gg'}(M^{d-p-1}) \qquad gg'=g''$
- Action: $U_g(M^{d-p-1})\,\mathcal{O}(C^p) = g(\mathcal{O})\,\mathcal{O}(C^p) \qquad g(\mathcal{O})$: (Group element)

[Continuous: $d*J_{p+1}=0, \quad Q=\int_{M^{d-p-1}} *J, \quad U_g=U_\alpha=e^{i\alpha\cdot Q}$]

○ For p>0 Screening by (p-1)-dim. Objects





Trivial Symmetry

In non-Abelian gauge theories with gauge group G

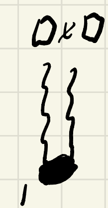
- The 1-form symmetry acts on fundamental Wilson Lines $\Gamma^{(1)} : U_\ell W_F = e^{2\pi i \ell/p} W_F$

$$W_F = e^{\oint i A}$$

G	$Z(G)$
$SU(N)$	$\mathbb{Z}_N$
$Sp(N)$	$\mathbb{Z}_2$
$Spin(N)$, N odd	$\mathbb{Z}_2$
$Spin(4N+2)$	$\mathbb{Z}_4$
$Spin(4N)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
E_6	$\mathbb{Z}_3$
E_7	$\mathbb{Z}_2$

$$p = \mathrm{ord}(Z(G))$$

$Z(G) :=$ Center



- Matter generically breaks to a subgroup by screening $SU(2N) + \Lambda^2$

In d>4 this could happen with non-perturbative states which become massless in the UV

[Cordova, Intriligator, Dumitrescu]

2-Group Primer:

- continuous flavor symmetry and continuous 1-form symmetry

$$F^{(1)} = U(1) \qquad J^{(1)} \text{ s.t } d * J^{(1)} = 0$$

$$\Gamma^{(1)} = U(1) \qquad J^{(2)} \text{ s.t } d * J^{(2)} = 0$$

- Action coupled to Background for the symmetries

$$S[A^{(1)}, B^{(2)}] = S + \int d^d x \left[A^{(1)} \wedge * J^{(1)} + B^{(2)} \wedge * J^{(2)} \right]$$

- 2-Group defined by

$$\begin{cases} A^{(1)} \to A^{(1)} + d\lambda^{(0)} \\ B^{(2)} \to B^{(2)} + d\Lambda^{(1)} + \frac{K}{2\pi} \lambda^{(0)} \, dA^{(1)} \end{cases}$$

$\lambda^{(0)}, \Lambda^{(1)}$ Transformation Parameters

- Bianchi Identity: $dH^{(3)} = \frac{K}{2\pi} dA^{(1)} \wedge dA^{(1)} \to H^{(3)} = dB^{(2)} + \frac{K}{2\pi} A^{(1)} \wedge dA^{(1)}$

- Example: Little string theory in 6d, no continuous 1-form symmetries in SCFTs

QUESTION: what if the 1-form symmetry is discrete?

(Center, Z(G), or subgroups thereof in Non-Abelian Gauge Theories)

- Discrete Generalization of Bianchi Identity: $\delta B_2 = A_1^* \Theta$

  - $B_2 \in C^2(M, \Gamma^{(1)})$ (2-Cochain) $\quad C^p \xrightarrow{\delta} C^{p+1}$
  - $A_1 : M \longrightarrow B\mathcal{F}$ (Classifying Space) $\quad A^*$ Pullback
  - Θ : Postnikov Class $\in H^3(B\mathcal{F}, \Gamma^{(1)})$

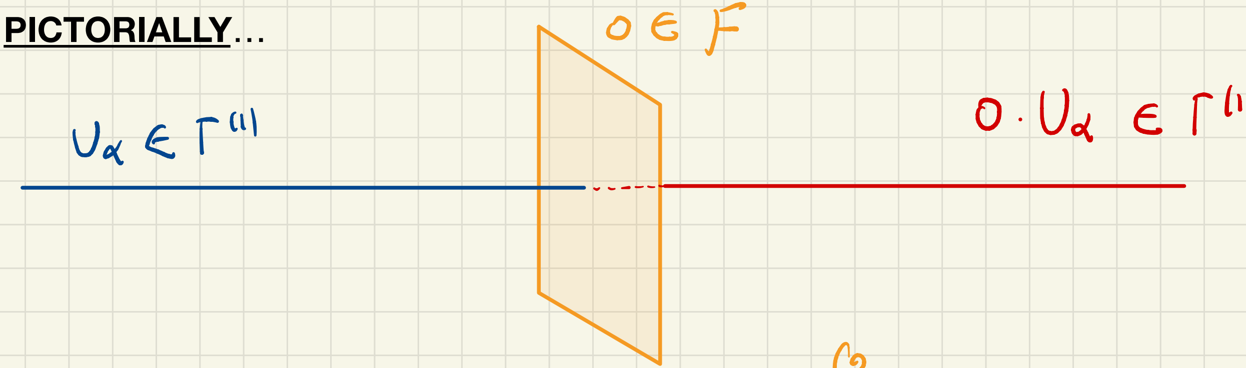
$\Theta = \mathrm{Boch}(w_2) + \ldots \quad w_2 \in H^2(B\mathcal{F}, \mathcal{Z})$ Obstruction: $\mathcal{F} = \frac{F}{\mathcal{Z}} \longrightarrow F$ (Cover)

Bockstein Homomorphism: $H^2(B\mathcal{F}, \mathcal{Z}) \longrightarrow H^3(B\mathcal{F}, \Gamma^{(1)})$

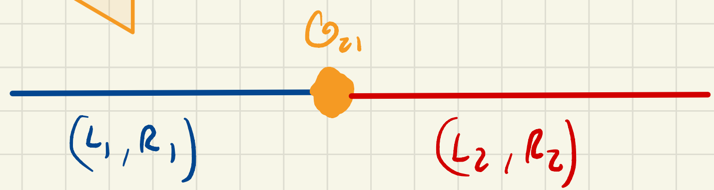
Associated to Short Exact Sequence: $0 \rightarrow \Gamma^{(1)} \rightarrow \mathcal{E} \rightarrow \mathcal{Z} \rightarrow 0$

- **INPUT**: Physics determines $\Gamma^{(1)}, \mathcal{E}, F, \mathcal{Z}$ Field Theory Spectrum or String Theory

PICTORIALLY...



In terms of Charged Operators:



$\mathcal{O}_{12}$ Non-Genuine Local Operator $\in R_2 \otimes R_1^*$ Of flavor symmetry algebra

L_1, L_2 Line defects in same Equivalence Class

- (L_1, R_1) (L_2, R_2) Same Equivalence Class $\hat{\mathcal{E}} = \mathrm{Hom}(\mathcal{E}, U(1))$

$\hat{\mathcal{E}}$ Ponryagin Dual of $\mathcal{E}$

2. 5d SU(2) theory

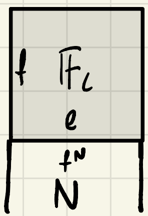
E_1 $(F = SU(2))$ 5d N=1 SCFT with rank 1 Coulomb Branch, which at low-energy is described by

$SU(2)_0$ N=1 Super Yang-Mills UV SCFT $\xrightarrow{m = 1/g^2_{YM}}$ IR $SU(2)_0$ SYM

- Engineered in M-Theory by Calabi-Yau: complex cone over $\mathbb{F}_2$.

<u>Smooth Calabi-Yau:</u>

$CY_3 =$



$\mathbb{F}_2$ Intersection data : $\begin{cases} e \cdot e = -2 \\ e \cdot f = 1 \\ f \cdot f = 0 \end{cases}$

$CY \Rightarrow N = \mathcal{O}(p) + \mathcal{O}(q) \quad p + q = -2$
$\forall$ curve

DICTIONARY

- $\mathbb{F}_2$ Hirzebrouch Surface, gauge symmetry
- N Non-compact surface, flavor
- $C_3 = A_{U(1)_N} \wedge \omega_2^{PD}(N) + A_{U(1)_g} \wedge \omega_2^{PD}(\mathbb{F}_2)$

  PD : Poincaré dual of surface
- $U(1)_N$ Cartan of $F = SU(2)$
- M2 on f W-Boson of $SU(2)_0$ $m_W = \mathrm{vol}(f)$
- $\Phi \sim \mathrm{vol}(\mathbb{F}_2)$ Coulomb branch scalar

- Extended Coulomb Branch $\phi, m_f, m_e > 0$ $\qquad G \times F = U(1)_g \times U(1)_N$

- Electric and center Charges given by intersection numbers:

$$q_{ee}(f) = -f \cdot \mathbb{F}_2 = 2 \qquad q_{ee}(e) = -e \cdot \mathbb{F}_2 = 0$$

$$q_N(f) = -f \cdot N = -1 \qquad q_N(e) = -e \cdot N = 2$$

	q_{ee}	q_N
e	0	2
f	2	-1

- Instanton Symmetry in the non-Abelian gauge theory $m_w = vol(f) = 0$

$$J_I = * \, Tr\, F_g \wedge F_g \qquad q_I = \frac{1}{2} q_{U(1)_N} + \frac{1}{4} q_{ee}$$

$e+f$ Instanton particle $\qquad q_{ee}(e+f) = 0 \qquad q_I(e+f) = 1$

SCFT symmetry is broken to U(1) instanton in the gauge theory by mass deformation

$$SU(2) \longrightarrow U(1)_I$$

- 1-Form symmetry: $\Gamma^{(1)} = \mathbb{Z}_2$

[Morrison, Schäfer-Nameki, Willett; Albertini, Del Zotto, Garcia Etxebarria, Hosseini]

Gauge theory: all BPS particle have electric charge 0 mod 2

Can be geometrically computed in M-Theory via Relative Cohomology (NON-COMPACT M2)

- Structure group which acts faithfully on the spectrum

$$S = \frac{G \times F}{\mathcal{E}} = \frac{U(1)_g \times U(1)_N}{\mathbb{Z}_4}$$

Generator: $\left(\frac{1}{4}, \frac{1}{2}\right) \in \left(\mathbb{R}/\mathbb{Z}, \mathbb{R}/\mathbb{Z}\right)$

- Coulomb branch of SCFT: locus where $\mathrm{vol}(e) = 0$ $\qquad F = SU(2)$

$$S = \frac{U(1)_g \times SU(2)}{\mathbb{Z}_4}$$

$$0 \to \Gamma^{(1)} = \mathbb{Z}_2 \to \mathcal{E} = \mathbb{Z}_4 \to Z = \mathbb{Z}_2 \to 0$$

$\Gamma^{(1)} : \left(\frac{1}{2}, 0\right) \in \left(\mathbb{R}/\mathbb{Z}, \mathbb{R}/\mathbb{Z}\right)$ $\qquad Z : \left(0, \frac{1}{2}\right) \in \left(\mathbb{R}/\mathbb{Z}, \mathbb{R}/\mathbb{Z}\right)$

- **2-group**: $\boxed{dB_2 = \mathrm{Bock}\left(w_2\left(SO(3)\right)\right) = w_3}$ where $F = SO(3) \cong SU(2)/\mathbb{Z}_2$

w_2 Is obstruction $SO(3) \to SU(2)$

3. 5d 2-Groups, perturbative vs Non-Perturbative

We have a 2-group when $$0 \to \Gamma^{(1)} = \mathbb{Z}_n \to \mathcal{E} = \mathbb{Z}_{2n} \to \mathcal{Z} = \mathbb{Z}_2 \to 0$$

- with $n = \text{even}$

- $n = \text{odd}$ $\quad \mathbb{Z}_{2n} = \mathbb{Z}_2 \times \mathbb{Z}_n$ (splits) $\quad \mathrm{Bock}(w_2) = 0$

We have just encountered an example of non-perturbative 2-group

- Perturbative 2-groups are realized in the gauge theory and NOT spoiled by BPS non-perturbative states

- Non-perturbative 2-groups are realized by the combined gauge theory and non-perturbative spectrum

Example of Perturbative 2-Group:

$SU(4)_2 + 3\Lambda^2$

$\Gamma^{(1)} = \gcd(4,2)$

- $k=2$ Coefficient of $k\, CS_5(A_g)$
- 3 Hypermultiplets in the 2-index antisymmetric Λ^2
- $F = Sp(3)_{pert} \times SU(2)_{INST}$ ($U(1)_I$ Enhances to $SU(2)_{Inst}$)
- $\Gamma^{(1)} = \mathbb{Z}_2$

- Coulomb Branch: $U(1)_1 \times U(1)_2 \times U(1)_3 \times Sp(3)_{pert} \times U(1)_I$

  $m(\Lambda^2) = 0$

- Center Charges:

	$q_{ee}^{(1)}$	q_{ee}^{2}	q_{ee}^{3}	$\mathbb{Z}_2 = Z(Sp(3))$	q_F
W^1	2	-1	0	0	0
W^2	-1	2	-1	0	0
W^3	0	-1	2	0	0
Inst	0	0	0	0	1
Λ^2	1	1	1	1	0

- $\Gamma^{(1)}$ gen. by $\left(\frac{1}{2}, 0, \frac{1}{2}, 0, 0\right)$
- $\mathcal{E}$ gen. by $\left(\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{2}, 0\right)$

Any Representation has an electric or center symmetry charge

- Structure Group: $S = \dfrac{SU(4) \times Sp(3)}{\mathbb{Z}_4} \times U(1)_I$

  $\mathcal{E} = \mathbb{Z}_4$ Acts as $\mathbb{Z}_2$ gen by $\left(0, 0, 0, \tfrac{1}{2}, 0\right)$ on $Sp(3)$

- Sequence: $0 \to \Gamma^{(1)} = \mathbb{Z}_2 \to \mathcal{E} = \mathbb{Z}_4 \to \mathcal{Z} = \mathbb{Z}_2 \to 0$

- 2-Group: $\boxed{\delta B_2 = \mathrm{bock}(w_2) = w_3}$ w_2 Is obstruction $Sp(3)/\mathbb{Z}_2 \to Sp(3)$

- Flavor: $\boxed{F = \dfrac{Sp(3)}{\mathbb{Z}_2} \times SU(2)}$

The instanton symmetry and the states charged under it are spectators

Example of non-Perturbative 2-Group:

- $SU(M)_M$ with $M=$ even $\quad \Gamma^{(1)} = \mathbb{Z}_M \,,\; \mathcal{E} = \mathbb{Z}_{2M} \,,\; \mathcal{F} = SO(3)$

- $SU(2m+2)_4 + 2\Lambda^2$ m even $\quad \Gamma^{(1)} = \mathbb{Z}_2 \quad \mathcal{E} = \mathbb{Z}_4 \times \mathbb{Z}_2$

  $\mathfrak{g}_F = SU(2)_N \times SU(2)_M \times SU(2)_P$ $\quad M$ Perturbative , $\quad N, P$ Non-Pert.

  $\mathcal{F} = SU(2)_N \times SO(3)_{NP} \times SO(3)_{MP}$ $\quad NP$ & MP Diagonal Combinations

  Non-trivial sequence and 2-Group $\quad 0 \to \mathbb{Z}_2 \to \mathbb{Z}_4 \to \mathbb{Z}_2^{NP} \to 0$

  $\mathbb{Z}_2^{NP} = Z(SO(3))_{NP}$ $\quad dB_2 = \mathrm{Bock}\left(B[SO(3)_{NP}], \mathbb{Z}_2\right)$

$\mathfrak{T}$	$\mathcal{O}_{\mathfrak{T}}$	$\mathcal{F}_{\mathfrak{T}}$	F'
$SU(2)_0$	$\mathbb{Z}_2$	$SO(3)$	$SU(2)$
$SU(2n)_{2n}$	$\mathbb{Z}_{2n}$	$SO(3)$	$SU(2)$
$Sp(2m-1)_0+\mathbf{\Lambda}^2$ $=SU(2m)_{m+4}+\mathbf{\Lambda}^2$	$\mathbb{Z}_2$	$SO(3)\times SU(2)$	$SU(2)\times SU(2)$
$Sp(2m)_0+\mathbf{\Lambda}^2$	$\mathbb{Z}_2$	$SO(3)\times SU(2)$	$SU(2)\times SU(2)$
$SU(2n)_4+2\mathbf{\Lambda}^2$	$\mathbb{Z}_2$	$SO(3)\times SO(3)$	$SU(2)\times SO(3)$
$SU(2n)_0+2\mathbf{\Lambda}^2$	$\mathbb{Z}_2$	$SO(3)\times SU(2)$	$SU(2)\times SU(2)$
$\mathrm{Spin}(4n)+(4n-3)\boldsymbol{F}$	$\mathbb{Z}_2$	$PSp(4n-2)$	$Sp(4n-2)$
$\mathrm{Spin}(2n+1)+(2n-3)\boldsymbol{F}$	$\mathbb{Z}_2$	$SO(3)\times Sp(2n-3)$	$SU(2)\times Sp(2n-3)$
$\mathrm{Spin}(4n+2)+(4n-2)\boldsymbol{F}$	$\mathbb{Z}_2$	$SO(3)\times PSp(4n-2)$	$\frac{SU(2)\times Sp(4n-2)}{\mathbb{Z}_2^{\mathrm{diag}}}$
$\mathrm{Spin}(4n)+(4n-4)\boldsymbol{F}$	$\mathbb{Z}_2$	$SO(3)\times PSp(4n-4)$	$SU(2)\times PSp(4n-4)$
$\mathrm{Spin}(4n+2)+4m\boldsymbol{F}$; $0\leq m\leq n-1$	$\mathbb{Z}_2$	$PSp(4m)$	$Sp(4m)$
$\mathrm{Spin}(4n{+}2){+}(4m{+}2)\boldsymbol{F}$; $0\leq m\leq n-2$	$\mathbb{Z}_2$	$PSp(4m+2)$	$Sp(4m+2)$
$SU(4)_2+\mathbf{\Lambda}^2$	$\mathbb{Z}_2$	$SO(3)$	$SU(2)$
$SU(4)_2+3\mathbf{\Lambda}^2$	$\mathbb{Z}_2$	$PSp(3)\times SU(2)$	$Sp(3)\times SU(2)$
$\mathrm{Spin}(7)+3\boldsymbol{F}$	$\mathbb{Z}_2$	$SO(3)\times Sp(3)$	$SU(2)\times Sp(3)$
$\mathrm{Spin}(12)+2\boldsymbol{S}$	$\mathbb{Z}_2$	$SO(3)^3$	$SU(2)^2\times SO(3)^2$

4. 6d Theories with 2-Groups and Classification

6d N=(1,0) Theories are quiver gauge theories at low-energy

G can be $\emptyset$ or E_n

— Bifundamental Hypers

- There are also tensor multiplets $\left(\phi^i, B^i_{\mu\nu} \ldots\right)$

$$S \supset \Omega_{ij} \int \phi^i \, \mathrm{Tr}\left(F^j \wedge * F^j\right) + \Omega_{ij} \int B^i \wedge \mathrm{Tr}\left(F^j \wedge F^j\right) \qquad H^i = dB^i + \ldots$$

- Tensor Branch $\langle \phi^i \rangle \neq 0$

Topological coupling necessary for reducible gauge anomaly cancelation via GSWS mechanism

$$dH^i = I_4^i$$

I_4^i  I_4^j

$$I_8^{AN} = \frac{1}{2} \Omega_{ij} I^i I^j$$

- BPS String Charge lattice, charged under B^i $\quad T_i \sim \Omega_{ij} \langle \phi^j \rangle \quad \langle Q^i, Q^j \rangle = \Omega_{ij} Q^i Q^j$

$\Omega_{ij} \in \mathbb{Z}$ Positive definite for SCFTs

$$\Omega = \begin{pmatrix} M_1 & -1 & 0 & 0 & \cdots \\ -1 & M_2 & -1 & & \\ 0 & -1 & M_3 & & \\ 0 & 0 & & \ddots & \\ \vdots & \vdots & & & \end{pmatrix}$$

$$[g_0] - \underset{M_1}{g_1} - \underset{M_2}{g_2} \cdots\cdots - \underset{M_N}{g_N} - [g_{N+1}]$$

- In order to understand whether 2-Groups are allowed we need to compute $\Gamma^{(1)}, \mathcal{E}, \mathcal{F}, \mathcal{Z}$

  By computing the charges under the center symmetries of gauge and flavor groups.

  These include **BPS string states**, which become massless at strong coupling

$$q_{ee}(\text{W-BOSONS}), \quad q_{ee}(\text{HYPERS}), \quad q_{ee}(\text{STRINGS})$$

$$q_{\text{FLAVOR}}(\text{W-BOSONS}), \quad q_{\text{FLAVOR}}(\text{HYPERS}), \quad q_{\text{FLAVOR}}(\text{STRINGS})$$

?

- Two methods:

  - Extrapolate them from Elliptic genus computation, example E-string $[SO(8)] - \overset{\emptyset}{1} - [SO(8)]$

    $q(\text{strings})$

    $$E_8 \to SO(8) \times SO(8)$$

    $$248 \to \supset 8_v, 8_c, 8_s$$

  - Check Dirac quantization of string lattice when instanton density fractionalizes [Apruzzi, Dierigl, Lin]

$$\Omega_{ij} \int B^i \wedge \mathrm{Tr}(F^j \wedge F^j) \xrightarrow{Z(G)\text{-bacground}} \mathrm{Tr}(F^j \wedge F^j) = C_2(F^j) + \alpha_G^j B_2^2$$

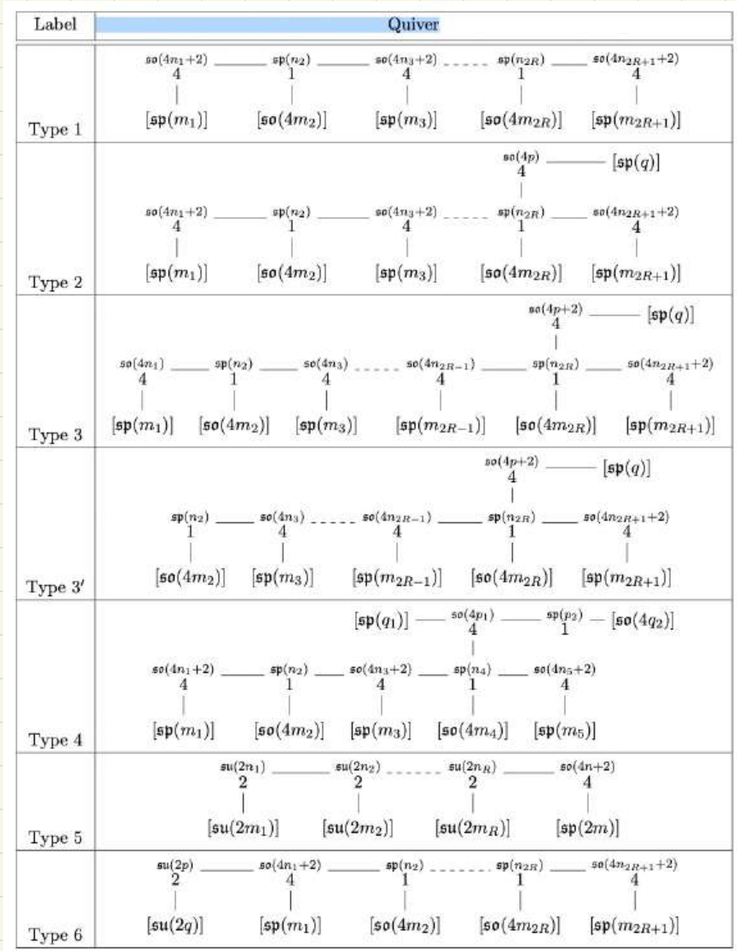
α_G Is fractional

Dirac quantization : $\Omega_{ij} Q^j \in \mathbb{Z} \Rightarrow \Omega_{ij} \alpha_G^j \in \mathbb{Z}$

Constrains the possible bakcgrounds

G	Z_G	α_G
$SU(N)$	$\mathbb{Z}_N$	$\frac{N-1}{2N}$
$Sp(N)$	$\mathbb{Z}_2$	$\frac{N}{4}$
$\mathrm{Spin}(2N+1)$	$\mathbb{Z}_2$	$\frac{1}{2}$
$\mathrm{Spin}(4N+2)$	$\mathbb{Z}_4$	$\frac{2N+1}{8}$
$\mathrm{Spin}(4N)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(\frac{N}{4}, \frac{1}{2})$
E_6	$\mathbb{Z}_3$	$\frac{2}{3}$
E_7	$\mathbb{Z}_2$	$\frac{3}{4}$

C L A S S I F I C A T I O N



$0 \to \Gamma^{(1)} = \mathbb{Z}_2 \to \mathcal{E} = \mathbb{Z}_4 \to \mathcal{Z} = \mathbb{Z}_2 \to 0$

$\delta B_2 = \mathrm{bock}(w_2)$

$\mathcal{F} = \left(\prod so \; \prod sp\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2$

$\mathcal{F} = \left(\prod so \prod sp\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2 \times \mathbb{Z}_2$

$\mathcal{F} = \left(\prod so \prod sp\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2 \times \mathbb{Z}_2$

$\mathcal{F} = \left(\prod so \prod sp\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2$

$\mathcal{F} = \left(\prod so \prod sp\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2$

$\mathcal{F} = \left(\prod su \times sp\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2$

$\mathcal{F} = \left(\left(\prod sp \prod so\right) \times su\right)/\mathbb{Z}_2 \qquad \Gamma^{(1)} = \mathbb{Z}_2$

Conclusions

- We have found 2-group symmetry involving a non-simply connected continuos flavor symmetry and a discrete 1-form symmetry in 5d SCFTs
- We computed the global structure of many 5d SCFTs
- We classified 6d SCFTs with such 2-group symmetries
- We provided general methods to compute the global structure of flavor symmetries