THE IR STRUCTURE OF CELESTIAL GLUON AMPLITUDES

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$\boxed{\text{MOTIVATION}}$ Flat Space Holography

$\Uparrow$

SYMMETRIES

$\boxed{\text{MOTIVATION}}$ Flat Space Holography

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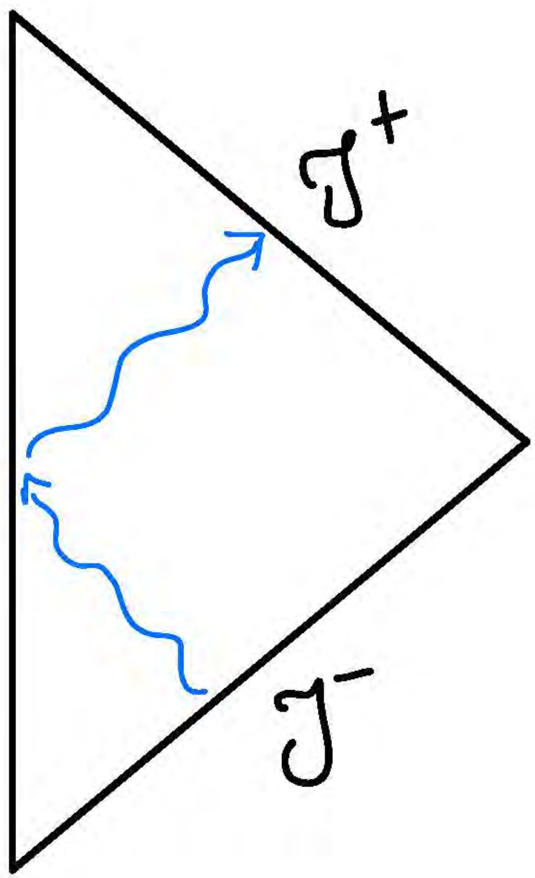
SYMMETRIES



- GRAVITY + BOUNDARY CONDITIONS AT $\mathcal{J}^{+/-}$

∞-DIMENSIONAL SYMMETRIES (B.M.S.) [1962] $\supset$ POINCARÉ

MOTIVATION

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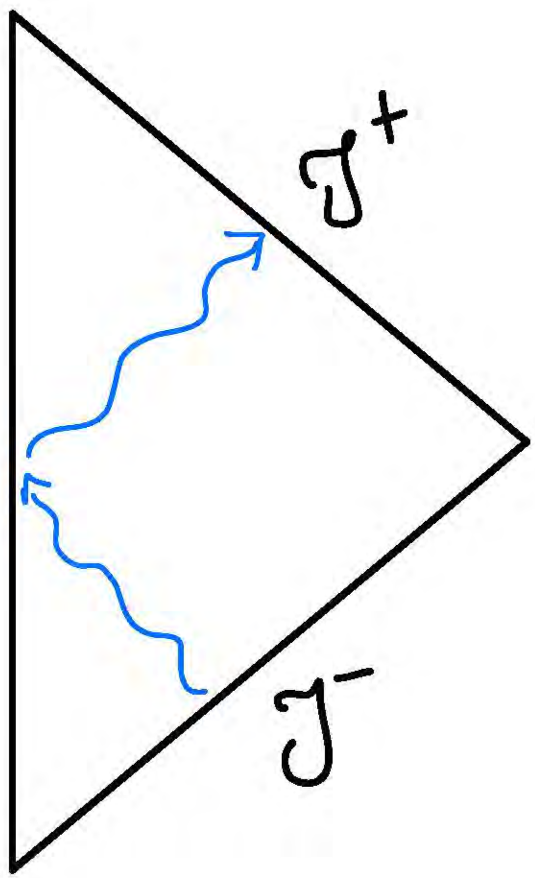

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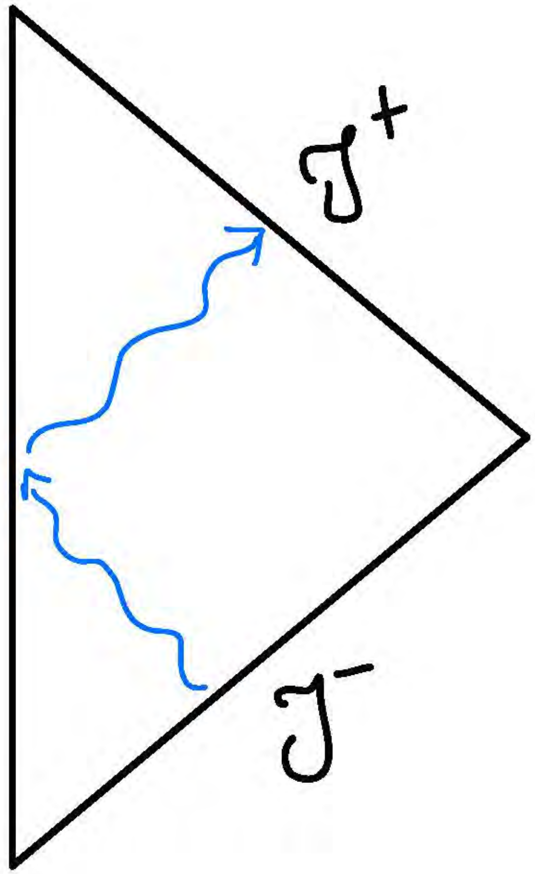

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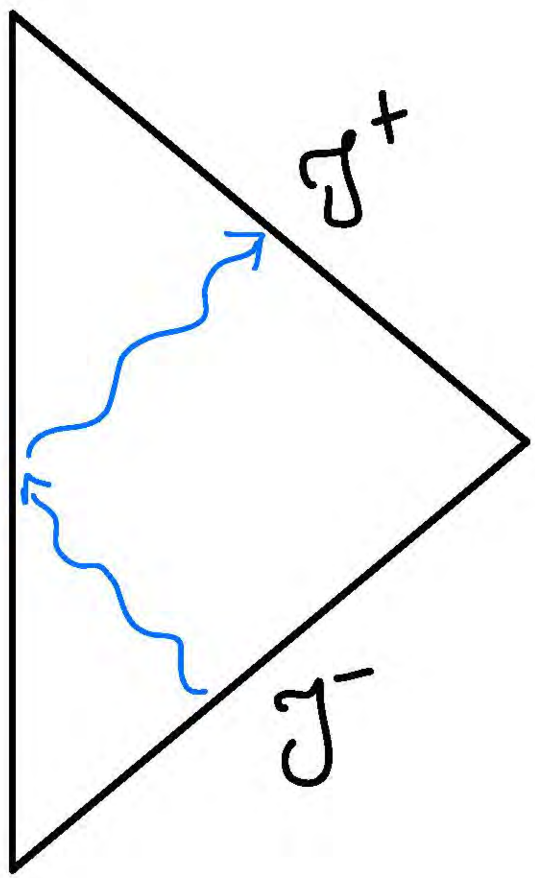

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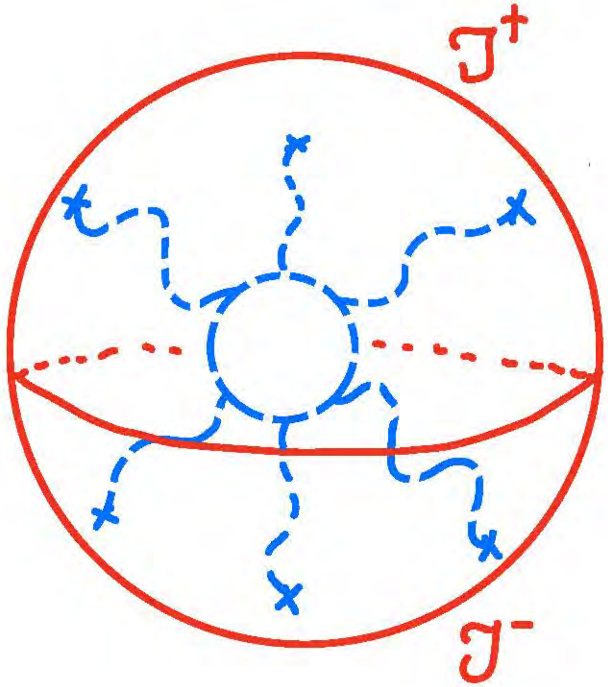
WHAT ARE THE ROLE OF THESE SYMMETRIES AT QUANTUM LEVEL?

S-MATRIX FOR MASSLESS PARTICLES



$$\langle \text{OUT} | \, S \, | \text{IN} \rangle$$

$\mathcal{I}^+ \qquad \mathcal{I}^-$

4D BULK / 2D CELESTIAL SPHERE

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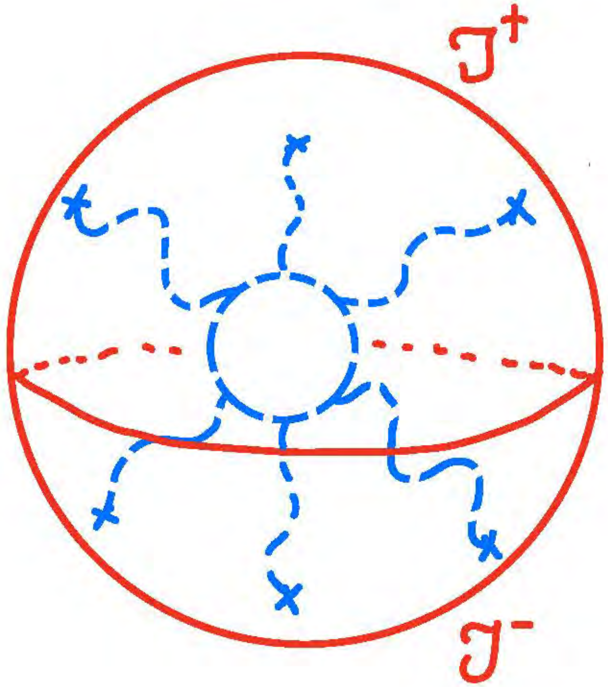

$$\begin{array}{ccc} \langle \text{OUT} | & S & | \text{IN} \rangle \\ \downarrow & & \downarrow \\ \mathcal{I}^+ & & \mathcal{I}^- \end{array}$$

4D BULK / 2D CELESTIAL SPHERE

WHAT ARE THE PROPERTIES OF THIS 2D THEORY?

OUTLINE

- Mellin Basis & Symmetries
- Celestial Factorization in IR Regulated Amplitudes
- CFT for IR divergences
- Conclusions

$\boxed{\text{A NEW BASIS}}$ [PASTERSKI & SHAO (2017)]

- USUAL BASIS $\rightsquigarrow$ PLANES WAVES $\psi_p(x) = e^{ip\cdot x}$

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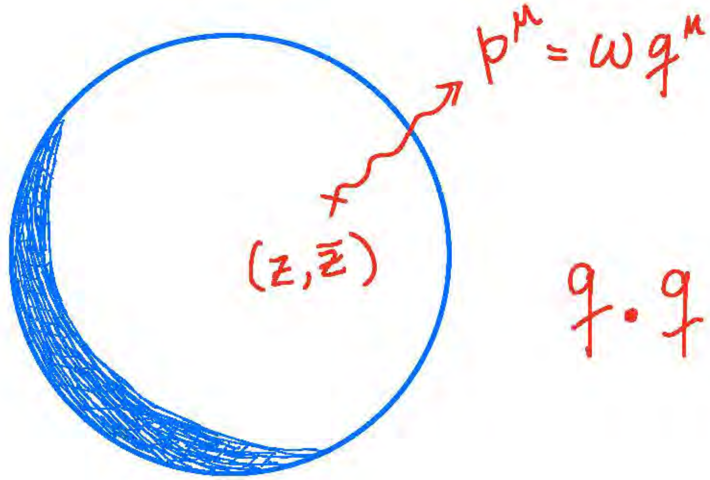

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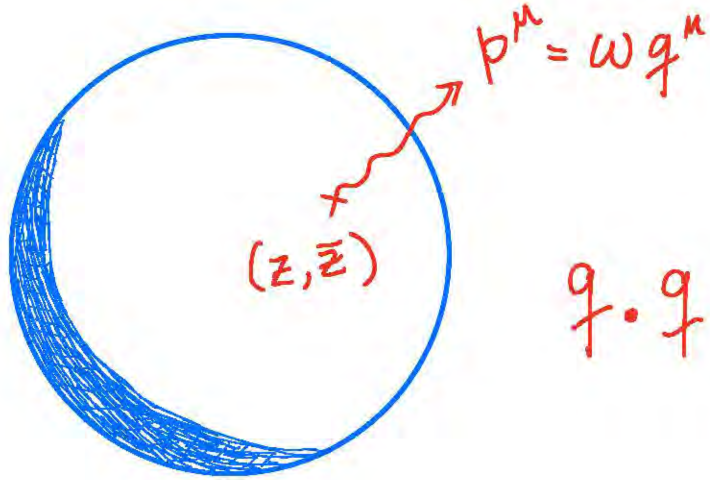

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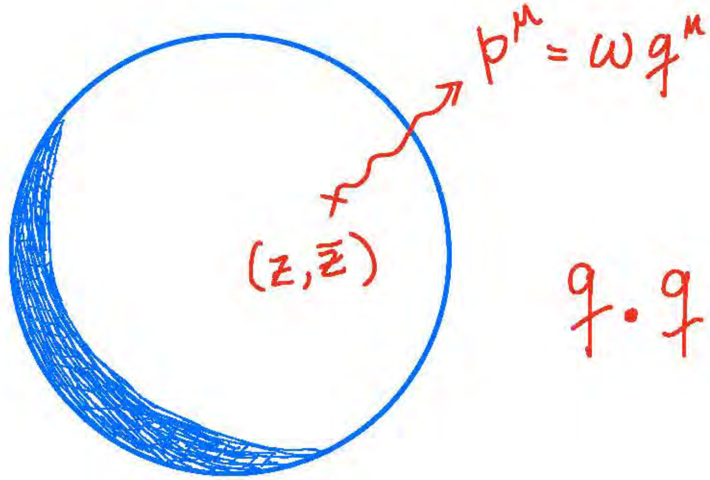
$$z \longrightarrow z' = \frac{az+b}{cz+d}$$

$$\begin{pmatrix} a & b \\ c & b \end{pmatrix} \in SL(2,\mathbb{C})$$

GLOBAL CONFORMAL SYMMETRY IN 2D



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- CHANGE OF BASIS USING **MELLIN TRANSFORM**

PLANE WAVE : $\exp(i\overset{\text{ENERGY}}{\omega}\,\underset{\text{POSITION ON CS}}{q}\cdot x)$

(SCALAR)

NEW STATE : $O_\Delta(z;x)$

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ENERGY (ω)

POSITION ON CS (q)

NEW STATE : $$O_\Delta(z;x) = \int_0^\infty \frac{d\omega}{\omega}\,\omega^\Delta\, e^{i\omega q\cdot x}$$

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- SUMMING OVER ALL ENERGY SCALES

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- SUMMING OVER ALL ENERGY SCALES
- EXCHANGING ENERGY (ω) BY SCALING DIMENSION (Δ)

CONFORMAL BASIS

- UNDER $SL(2,\mathbb{C})$: TRANSFORMS AS A PRIMARY FIELD

$$\mathcal{O}_\Delta\left(\Lambda^\mu{}_\nu x^\nu, \frac{az+b}{cz+d}\right) = |cz+d|^\Delta \, \mathcal{O}_\Delta(x^\mu; z)$$

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- DELTA - NORMALIZABLE $\quad \left(\Psi_p(x), \Psi_{p'}(x)\right) = (2\pi)^3 (2p^0)\, \delta^{(3)}(p-p')$

SAME NORMALIZATION ON CONFORMAL BASIS $\rightarrow$ $\Delta = 1 + i\lambda$

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DIFFERENT KIND OF CFT

CELESTIAL AMPLITUDES

[PASTERSKI, SHAO (2017)]
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- IN MOMENTUM BASIS:

$$\mathcal{A}_n(\{p_i,\epsilon_j\}) = i(2\pi)^4\,\delta\Big(p_1+p_2-\sum_{k=3}^{n} p_k\Big)\,A(\{p_i,\epsilon_i\})$$

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- $\widetilde{\mathcal{A}}$ INVOLVES ARBITRARILY HIGH ENERGY PARTICLES (ω_k)

CELESTIAL AMPLITUDE = CORRELATION FUNCTION 2D - CFT

$$\boxed{\tilde{\mathcal{A}}_{\{\Delta_k, \ell_k\}}(\{z_k, \bar{z}_k\}) = \langle \mathcal{O}_{\Delta_1 \ell_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{\Delta_n \ell_n}(z_n, \bar{z}_n) \rangle}$$

TRANSFORMATION PROPERTIES

- UNDER $SL(2,\mathbb{C})$ LORENTZ TRANSFORMATION

$$\tilde{\mathcal{A}}_{\{\Delta_k,\ell_k\}}\left(\left\{\frac{a z_k+b}{c z_k+d},\ \frac{\bar{a}\bar{z}_k+\bar{b}}{\bar{c}\bar{z}_k+\bar{d}}\right\}\right)=\prod_{k=1}^{n}\left((c z_k+d)^{\Delta_k+\ell_k}(\bar{c}\bar{z}_k+\bar{d})^{\Delta_k-\ell_k}\right)\tilde{\mathcal{A}}_{\{\Delta_k,\ell_k\}}\left(\{z_k,\bar{z}_k\}\right)$$

$$h_k=\frac{\Delta_k+\ell_k}{2} \qquad \bar{h}_k=\frac{\Delta_k-\ell_k}{2}$$

FOR EACH EXTERNAL PARTICLE

ℓ_k : HELICITY

GLUON AMPLITUDES AT LOOP LEVEL

- MELLIN AMPLITUDES COMBINES HIGH AND LOW ENERGY STATES.
- THEORIES WITH MASSLESS EXCITATIONS (PHOTONS, GLUONS, ...) SUFFERS FROM IR RADIATION AT LOOP LEVEL.

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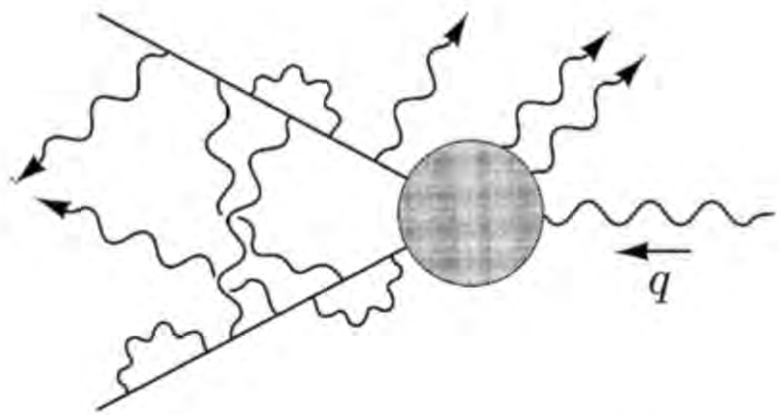

IR PROBLEM : ∞ INTERNAL EXCHANGES WITH VERY LOW ENERGY

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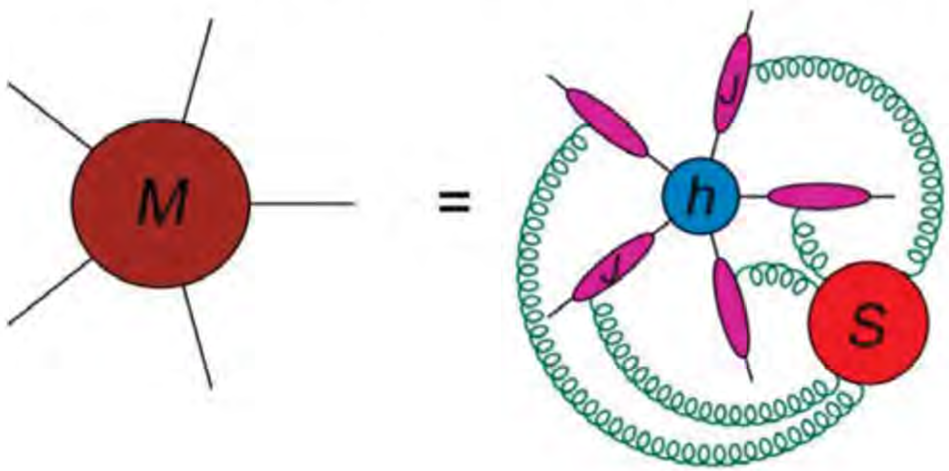

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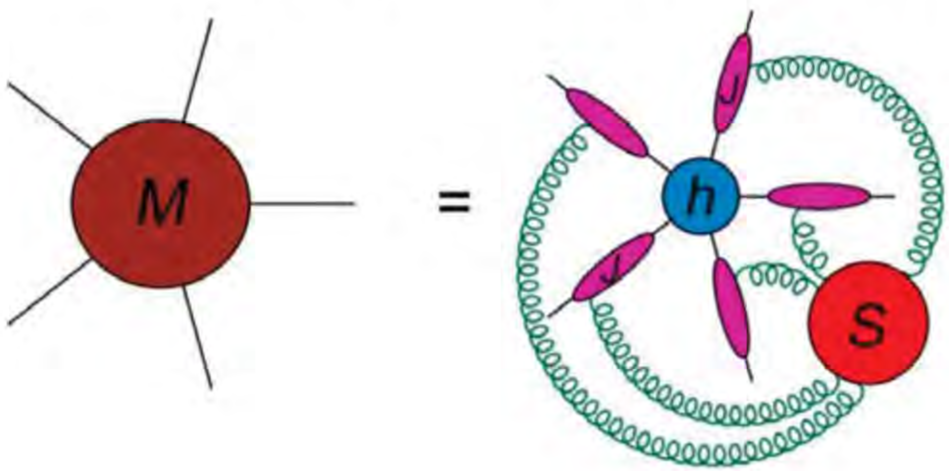

$$\mathcal{M} = \underbrace{\left(\prod_{i=1}^{n} J_i\right) S}_{\text{IR RESSUMATION}} \mathcal{H}$$

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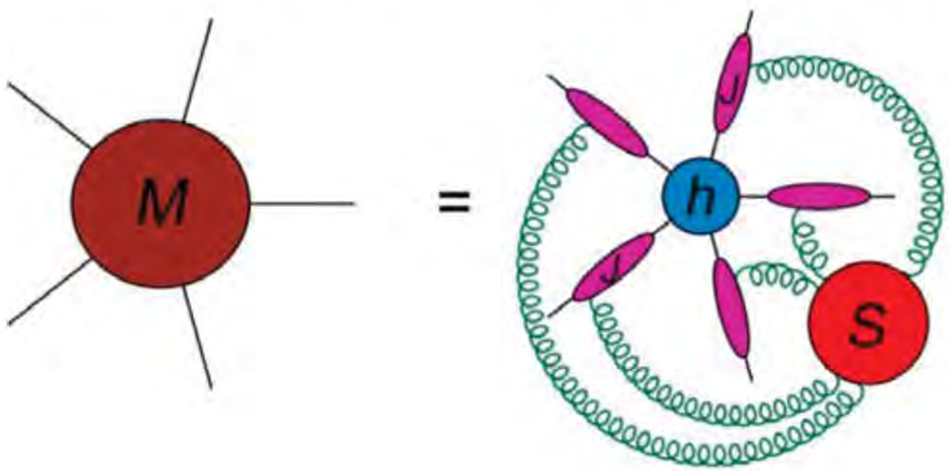

HARD PIECE

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CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

- WE CONSIDER A PROCESS INVOLVING n EXTERNAL GLUONS WITH HELICITIES $\ell_i = \pm$ AND SU(N) INTERNAL SYMMETRY

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- DIMENSIONAL REGULARIZATION $D = 4 - 2\epsilon$ $\epsilon < 0$ FOR INTERNAL INTERACTIONS

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CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

n-POINT AMPLITUDES $\mathcal{M}_n(p_1, \cdots, p_n) = \underbrace{\mathcal{Z}_n}_{\substack{\text{DIVERGENT} \\ \text{FOR } \Lambda \to 0}} \times \underbrace{\mathcal{A}_n}_{\text{FINITE}}$

EVOLUTION EQUATIONS $\frac{d}{d \log \mu^2} \mathcal{Z}_n = - \Gamma_n \mathcal{Z}_n$

ENERGY SCALE

ANOMALOUS DIMENSION

$$\mathcal{Z}(\mu^2, p_i \cdot p_j) = \mathcal{P} \exp \left\{ -\frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \Gamma_n (p_i \cdot p_j, \lambda^2) \right\}$$

$\boxed{\text{CELESTIAL } n\text{-POINT GLUON AMPLITUDE (ALL LOOPS)}}$

MAIN OBJECTIVE :

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

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CELESTIAL CFT BEHIND CORRELATOR

$$\mathbb{Z}_n(\mu^2, p_i \cdot p_j)$$

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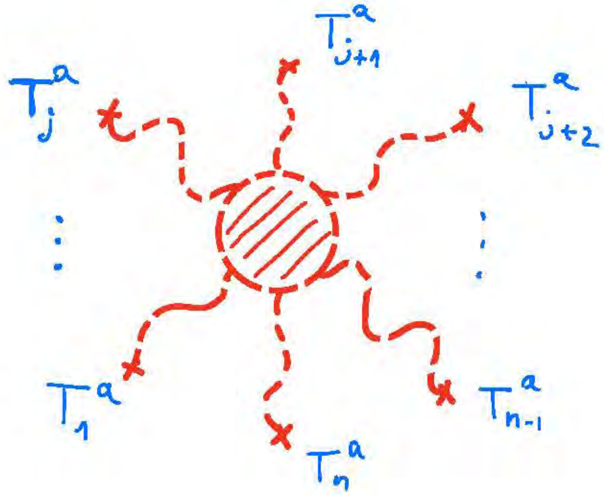
$$\left.\begin{array}{l} 1)\ N \gg 1 \\ \\ 2)\ \begin{array}{c}\text{FINITE } N \\ \text{TRUNCATED IN } \alpha_s\end{array} \end{array}\right\} \quad \begin{array}{c}\text{COLORED} \\ \text{FREE SCALAR}\end{array}$$

[MAGNEA 2021]

SU(N) GLUON AMPLITUDES (COLOR SPACE FORMALISM CATANI [1998])



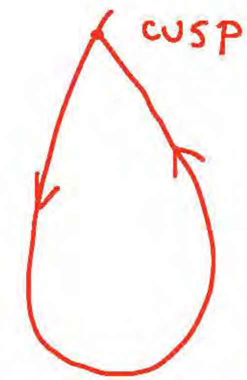
T_i^a : COLOR CHARGE OPERATOR IN ADJOINT REPRESENTATION

$\sum_{i=1}^{n} T_i^a = 0$: COLOR CHARGE CONSERVATION

IMPORTANT CONSTRAINT IN QCD AMPLITUDES: GARDI-MAGNEA BECHER-NEUBERT [2009]

$$\boxed{\sum_{j\neq i} \frac{\partial}{\partial \log \rho_{ij}} \Gamma_n^{(\mathrm{SOFT})} = \gamma^{i}_{cusp}(\alpha_s)\, \mathbb{1}}$$

ρ_{ij} invariant under $p_i \to \lambda_i p_i$



$$\left\langle \mathrm{Tr}\, \mathcal{P} e^{i g_s \oint A} \right\rangle$$

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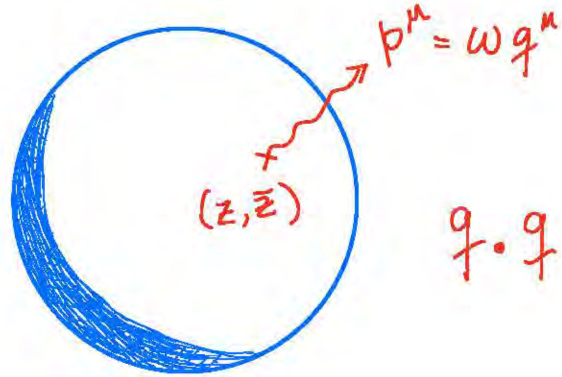

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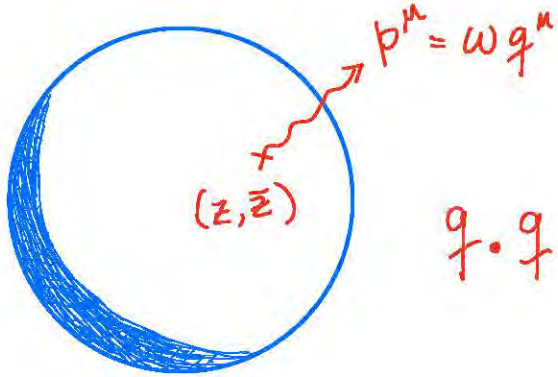
RECALL:



$q \cdot q = 0$

GARDI-MAGNEA
BECHER-NEUBERT
[2009]

$$\boxed{\sum_{j \neq i} \frac{\partial}{\partial \log \rho_{ij}} \Gamma_n^{(\text{SOFT})} = \gamma_{\text{cusp}}^i(\alpha_s)\, \mathbb{1}}$$

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

- MELLIN AMPLITUDE FACTORIZES AS WELL:

$$\tilde{\mathcal{A}}_n^{\text{ALL-LOOPS}} = Z_n(\mu^2, |z_{ij}|^2)\, \tilde{\mathcal{A}}_{\text{finite}}\left(h'_i = h_i - \tfrac{1}{4}K\right)$$

- INTERESTING PHYSICS ENCODED IN Z_n SPECIFIC THEORY?

IT CAN BE WRITTEN IN TERMS OF n-POINT CORRELATORS OF VERTEX OPERATORS $V_i(z, \bar{z})$ WITH CONFORMAL DIMENSION $\frac{1}{4}K$

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

- MELLIN AMPLITUDE FACTORIZES AS WELL:

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IT CAN BE WRITTEN IN TERMS OF n-POINT CORRELATORS OF VERTEX OPERATORS $\mathcal{V}_i(z,\bar{z})$ WITH CONFORMAL DIMENSION $\frac{1}{4}K$

$$\mathcal{Z}_n = \langle \mathcal{V}_{T_1} \otimes \mathcal{V}_{T_2} \otimes \cdots \otimes \mathcal{V}_{T_n} \rangle$$

2D-CFT CORRELATOR

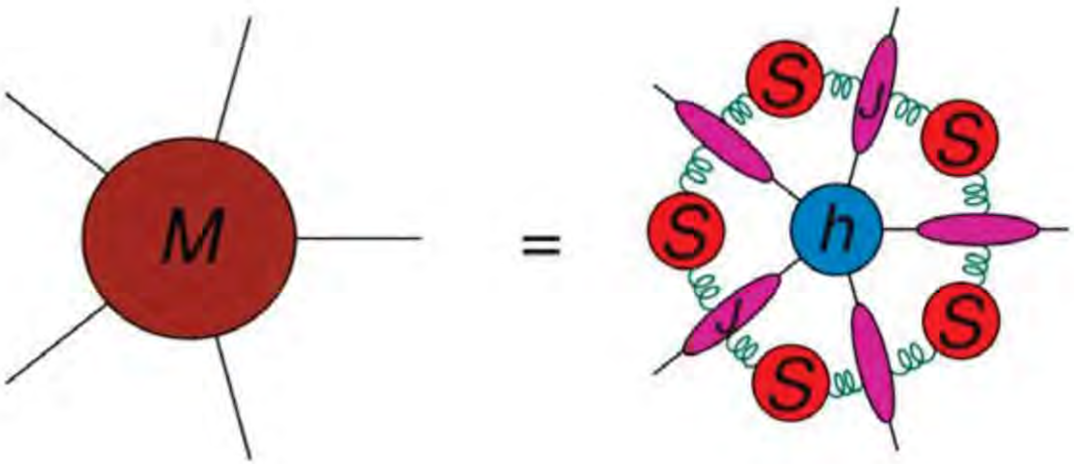
GARDI, SMILLIE, WHITE [2011, 2013]

$$K = \frac{1}{2}\int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2}\, \gamma_{cusp}(\alpha_s(\lambda^2))$$

DIVERGES WITH THE REGULATOR ϵ

LARGE N

$$\mathcal{Z}_n = \exp\left(-\frac{Nk}{4}\sum_{i<j}\overbrace{\left(\delta_{i,j+1}+\delta_{j,i+1}\right)}^{\text{NEAREST NEIGHBOUR INTERACTIONS}}\log\left(|z_{ij}|^2\right)\right)$$



- ONLY PLANAR DIAGRAMS CONTRIBUTE
- INTERACTIONS BETWEEN NEIGHBOURING LEGS

LARGE N

$$Z_n = \exp\left(-\kappa \sum_{i<j} (\delta_{i,j+1} + \delta_{j,i+1}) \log(|z_{ij}|^2)\right)$$

$4\kappa = N K = \text{fixed}$

$N \to \infty$

$$\boxed{\text{LARGE } N}$$

$$Z_n = \exp\left(-K \sum_{i<j} \left(\delta_{i,j+1} + \delta_{j,i+1}\right) \log\left(|z_{ij}|^2\right)\right)$$

$4\mathcal{K} = NK = \text{fixed}$

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COULOMB GAS DESCRIPTION :

LARGE N

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$N \to \infty$

COULOMB GAS DESCRIPTION : $\langle \phi^a(z)\, \phi^b(0) \rangle = -\mathcal{K}\, \delta^{ab} \log(|z_{ij}|^2)$

FREE BOSON WITH A COLOR INDEX

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NO LONGER AN OPERATOR $\longrightarrow$ T_i^a

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POSITION i POSITION $i+1$

NO LONGER AN OPERATOR $\longrightarrow$ $T_i^a = \underbrace{(0, \dots, 1, -1, 0, \dots, 0)}_{N^2 \to \infty \text{ COMPTES}} \rightsquigarrow \sum_{i=1}^{n} T_i^a = 0$

LARGE N

$$\mathbb{Z}_n = \left\langle :e^{iT_1\cdot\phi}: \; :e^{iT_2\cdot\phi}: \cdots :e^{iT_n\cdot\phi}: \right\rangle = \exp\left[-\sum_{i<j} \langle \phi\cdot T_i , \phi\cdot T_j \rangle \right]$$

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$$\boxed{V_{T_i} = e^{iT_i\cdot\phi}} \rightsquigarrow \boxed{I(\phi^a) = \frac{1}{8\pi K}\int d^2z\, \bar\partial\phi^a \partial\phi^a}$$

VERTEX OPERATOR

EFFECTIVE ACTION

$$\boxed{\text{LARGE N}}$$

$$\mathbb{Z}_n = \left\langle :e^{iT_1\cdot\phi}: \; :e^{iT_2\cdot\phi}: \cdots :e^{iT_n\cdot\phi}: \right\rangle = \exp\left[-\sum_{i<j} \langle \phi\cdot T_i, \phi\cdot T_j \rangle\right]$$

$$\boxed{V_{T_i} = e^{iT_i\cdot\phi}}$$

VERTEX OPERATOR

FINITE N

Γ_n AT FINITE N AND TWO-LOOP TRUNCATION

$$\mathcal{Z}_n = \exp\left(\frac{K}{2} \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \log\left(|z_{ij}|^2\right)\right)$$

ALSO ADMITS A COLORED FREE SCALAR REP.

[MAGNEA 2021]

FINITE N : BEYOND DIPOLAR APPROXIMATION

$$\mathbb{Z}_n = \mathcal{P}\exp\left\{-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\Gamma_n\left(|z_{ij}|^2,\lambda^2\right)\right\}$$

Γ_n ANOMALOUS DIMENSION CONTRIBUTIONS

Kinematic Dependence	Tensor Structure	Loop order	Reference
$\frac{z_{ij}\,z_{kl}}{z_{il}\,z_{jk}}$ SL(2,C) INVARIANT RATIOS	$f_{abe}\,f^{e}{}_{cd}\,\{T_i^a, T_j^b\}\,T_j^b\,T_k^c$	3	ALMELID, DUHR, GARDI, MCLEOD WHITE [2016] [2017]
$\log(\|z_{ij}\|^2)$	$d_{abcd}\,\mathrm{tr}(T_i^a T_j^b T_k^c T_l^d)$	4	BECHER NEUBERT [2019]

FINITE N : BEYOND DIPOLAR APPROXIMATION

- CCFT CONTAINING NON-ABELIAN STRUCTURE

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  ↳ BOUNDARY ACTIONS FROM YM SYMPLECTIC SYMMETRIES ?

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- IR SAFE CELESTIAL AMPLITUDES $\mathcal{O}^{\text{BARE}}_{(\Delta,\ell)} = \mathcal{V}_i\, \mathcal{O}^{\text{DRESSED}}_{(\Delta,\ell)}$

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  ↳ FADDEEV - KULISH CONSTRUCTION

CONCLUSION

- NEW KIND OF HOLOGRAFY:

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CONCLUSION

- NEW KIND OF HOLOGRAFY:

$$\boxed{\text{CELESTIAL AMPLITUDES} \longleftrightarrow \text{2D-CFT}}$$

- SU(N) GAUGE THEORY, CELESTIAL AMPLITUDE SPLITS INTO HARD AND INFRARED DIVERGENT PIECES.

- CFT IDENTIFICATION FOR INFRARED DIVERGENCES

$$\mathbb{Z}_n = \langle \mathcal{V}_{T_1} \otimes \mathcal{V}_{T_2} \otimes \cdots \otimes \mathcal{V}_{T_n} \rangle$$

- LARGE N
- FINITE N TRUNCATED

$\mathcal{V}_i = e^{i T_i \cdot \phi}$ PRIMARIES $\phi^a \rightsquigarrow$ COLORED SCALAR

THANKS!

$\boxed{\text{FINITE } N}$

WZW SATISFIES KNIZHNIK - ZAMOLODCHIKOV EQUATIONS

$$\boxed{\left(\frac{\partial}{\partial z_i} - \frac{K}{2} \sum_{i \neq j} \frac{T_i \cdot T_j}{z_i - z_j} \right) \mathbb{Z}_n = 0}$$

IN OUR CASE THERE IS AN EXTRA TERM ON THE RIGHT HAND SIDE

$\mathbb{Z}_n$ NOT COMPATIBLE WITH WZW

CONFORMAL BASIS

[STIEBERGER & TAYLOR 2019]

- UNDER SPACE-TIME TRANSLATION : $\hat{P}^{\mu} = \hat{\omega}\, q^{\mu}(z,\bar{z})$

$$\delta_{\hat{P}^{\mu}} O_{\Delta}(z;x) = q^{\mu} \int_0^{\infty} \frac{d\omega}{\omega}\, \omega^{(\Delta+1)}\, e^{i\omega q\cdot x}$$

$$\propto O_{\Delta+1}(z;X)$$

IN PARTICULAR $\quad \hat{P}^{+} \equiv \hat{P}^{0} + \hat{P}^{3}$

$$\delta_{P^{+}} O_{\Delta} = O_{\Delta+1}$$

TRANSLATION IS A SHIFT IN CONFORMAL DIMENSION

CONFORMAL BASIS : SUMMARY

$\mathcal{O}_\Delta(z;X) \longrightarrow$ PRIMARY STATE UNDER LORENTZ TRANSFORMATIONS

$\downarrow$

SCALING DIMENSION : $\Delta = 1 + i\lambda \rightsquigarrow$ $SL(2,\mathbb{C})$ PRINCIPAL SERIES?

NOT SO CLEAR AS TRANSLATIONS RELATE $\mathcal{O}_\Delta$ with $\mathcal{O}_{\Delta+1}$