Aspects of the black hole/string transition

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Based on arXiv:2109.08563 with Juan Maldacena and Edward Witten

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[Horowitz, Polchinski '96, …]

Comparison between a black hole and a highly excited string

We will be considering string theory in $R^{1,D-1} \times \text{internal}$ $\quad d = D - 1$

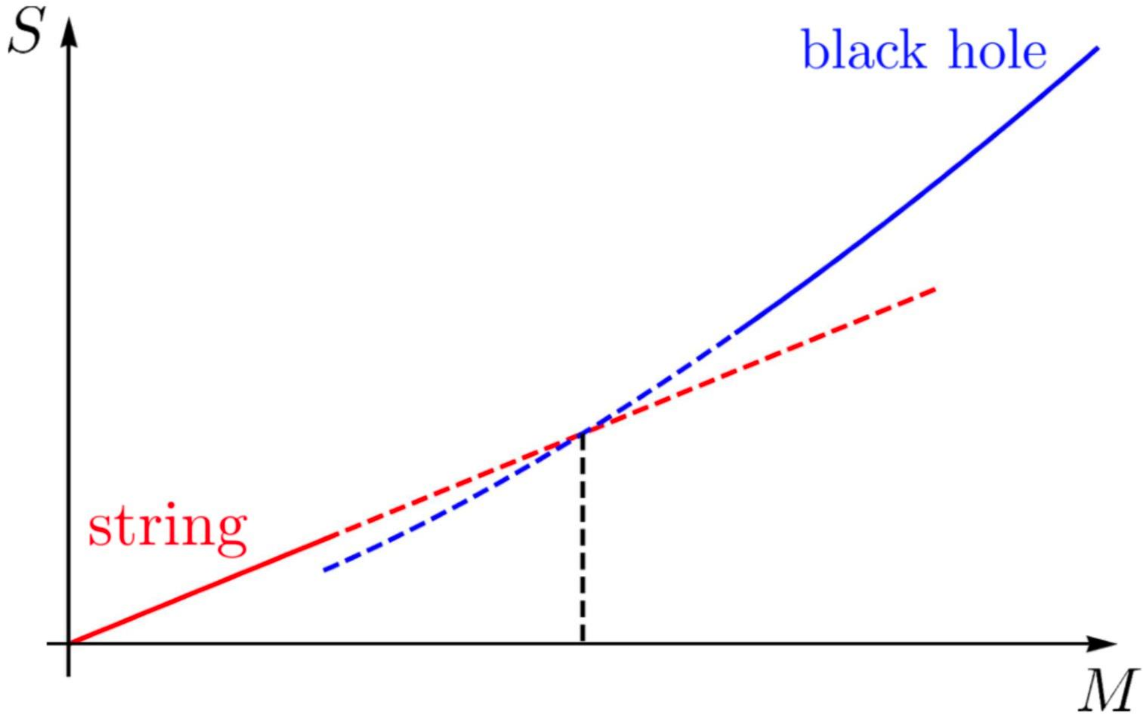
We will look at the Schwarzschild black hole.

It has long been speculated that the black hole will turn into a highly excited string when it reaches string size.

Comparison between a black hole and a highly excited string

Black hole: $S \propto r_s M \quad (S \sim r_s^{D-2},\ M \sim r_s^{D-3})$ $\qquad R^{1,D-1} \times \text{internal}$

String: $S = \beta_H M, \quad \beta_H \sim l_s$ $\qquad d = D - 1$



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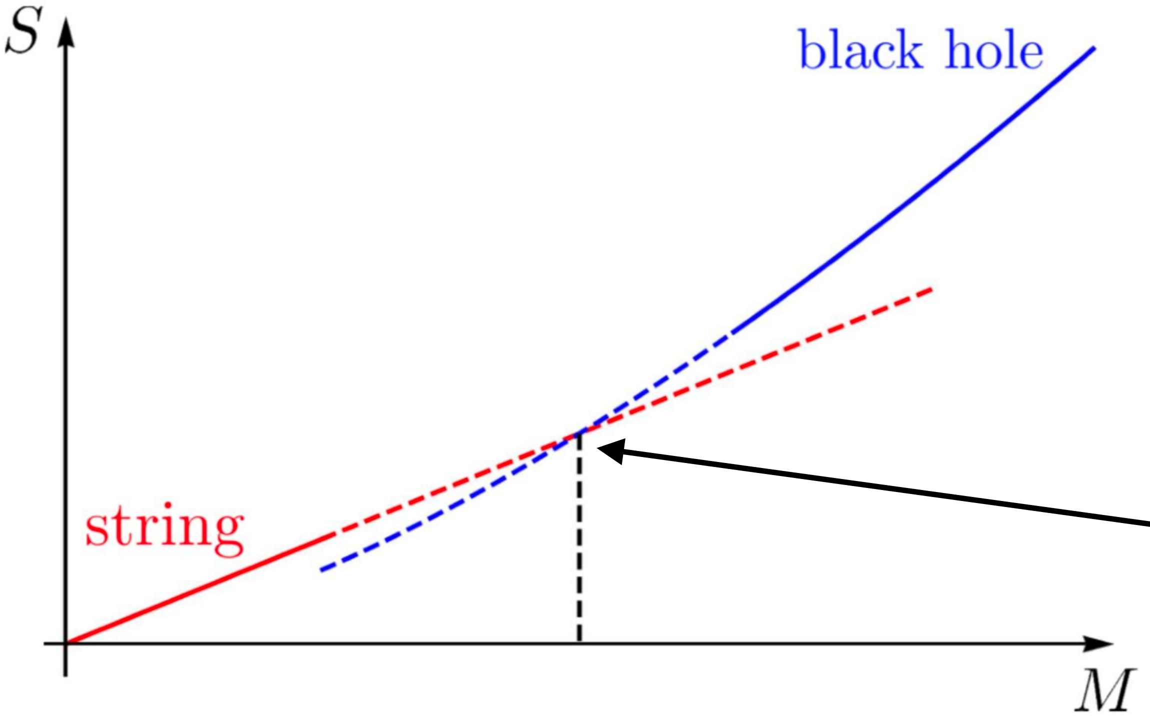

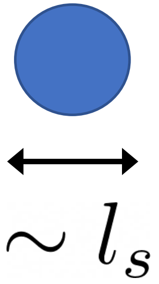

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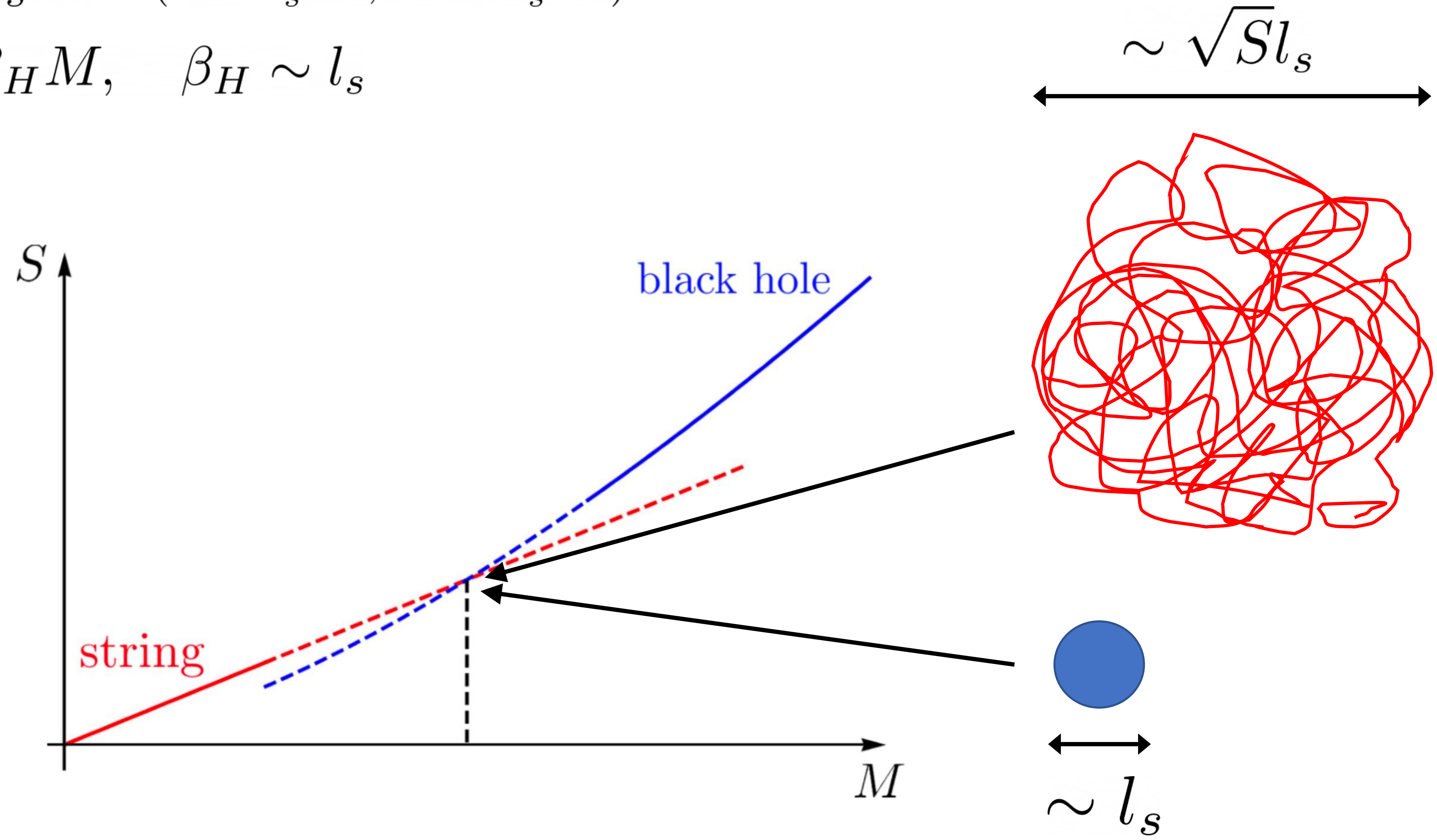


Hagedorn temperature and winding mode

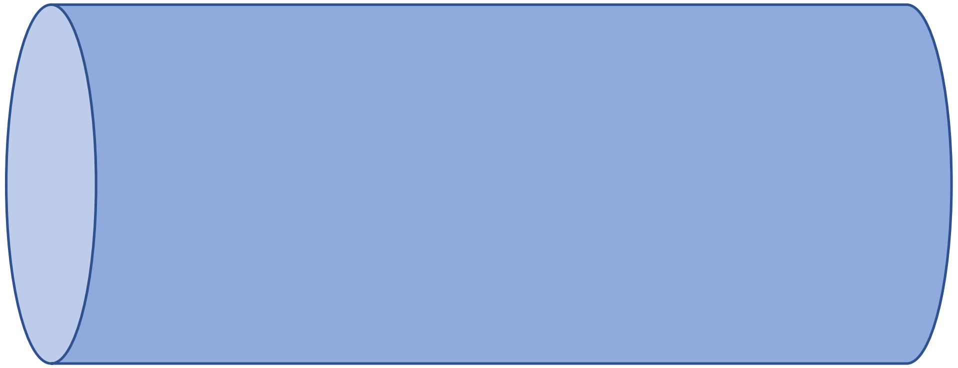
$\beta_H = \# l_s$ is called the (inverse) Hagedorn temperature and $\#$ is order one.

Density of states of a single oscillating string: $\rho(E) \propto e^{\beta_H E}$

$$z(\beta) = \int dE\, \rho(E) e^{-\beta E}$$

Spacetime perspective:

Euclidean time circle β



$S^1 \times R^d$

Hagedorn temperature and winding mode

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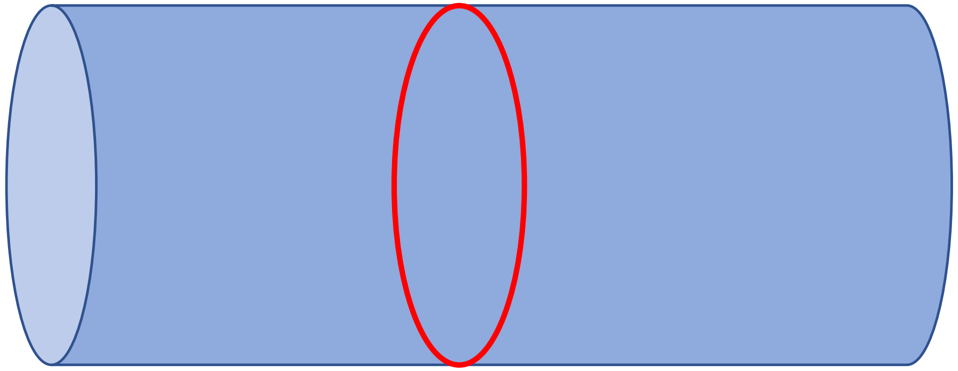

$S^1 \times R^d$

winding mode: $m^2 \propto \beta^2 - \beta_H^2$ (Bosonic, Type II)

As a side note, in the Heterotic string theory, the winding mode also carries some momentum.

Hagedorn temperature and winding mode

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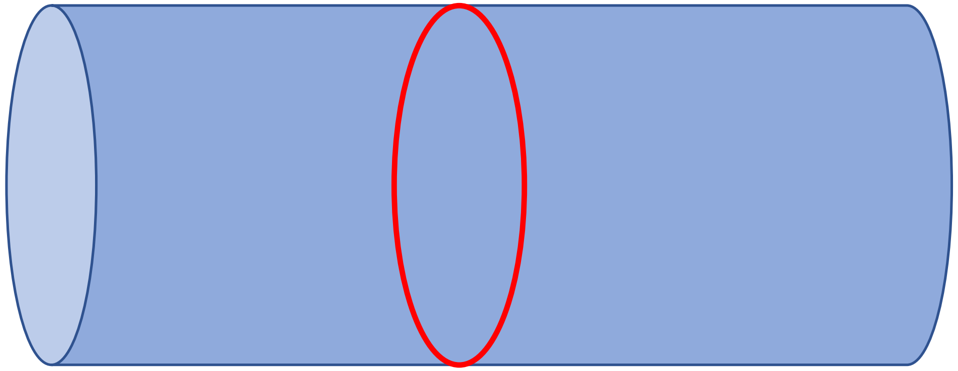

$S^1 \times R^d$

winding mode: $m^2 \propto \beta^2 - \beta_H^2$ (Bosonic, Type II)

We can describe it via a complex scalar field in d dimensions

$$\int d^d x \, \left(|\nabla \chi|^2 + m^2 |\chi|^2 \right)$$

The field becomes light when we approach the Hagedorn temperature.

[Horowitz, Polchinski ’97]

A string star solution by Horowitz and Polchinski

We saw that the size of a string is much larger than the black hole with the corresponding mass.

However, gravitational attraction was neglected in this analysis.

A string star solution by Horowitz and Polchinski

Consider the effective theory when we are close to the Hagedorn temperature

$\beta - \beta_H \ll l_s$

The massless fields include gravity, dilaton and the B field (not excited).
There is also a nearly massless field: χ

$$I_d = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} e^{-2\phi_d} \left[-\mathcal{R} - 4(\nabla \phi_d)^2 + (\nabla \varphi)^2 + |\nabla \chi|^2 + m^2(\varphi)|\chi|^2 \right]$$

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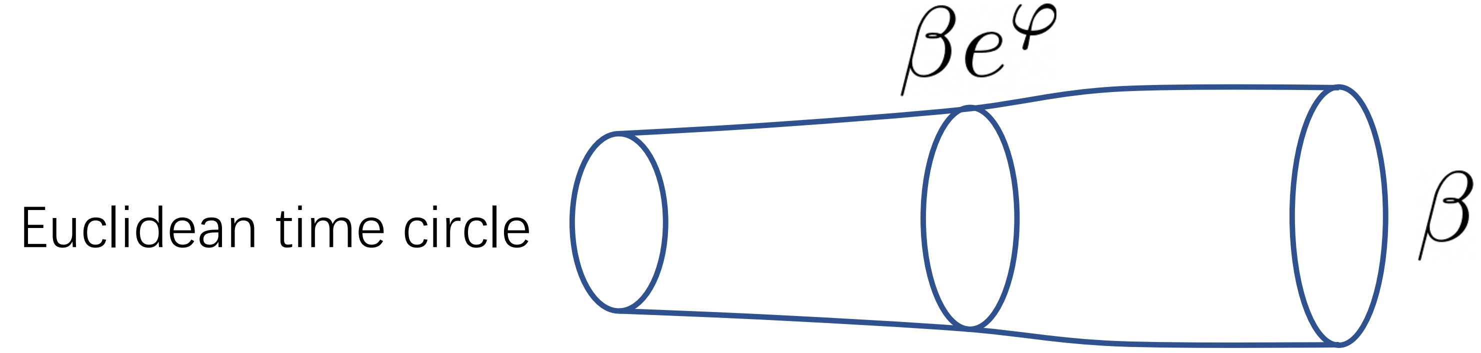
"radion"



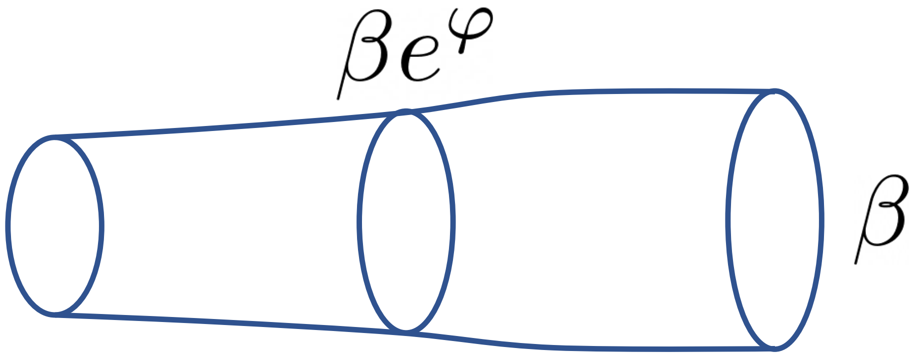
A string star solution by Horowitz and Polchinski

We look for a localized condensate solution.

$$I_d = \frac{1}{16\pi G_N}\int d^dx\sqrt{g}e^{-2\phi_d}\left[-\mathcal{R} - 4(\nabla\phi_d)^2 + (\nabla\varphi)^2 + |\nabla\chi|^2 + m^2(\varphi)|\chi|^2\right]$$

$$m^2(\varphi) = m_\infty^2 + \frac{\kappa}{\alpha'}\varphi + o(\varphi^2)$$

$\propto (\beta^2 - \beta_H^2)$



Reduce to equations:

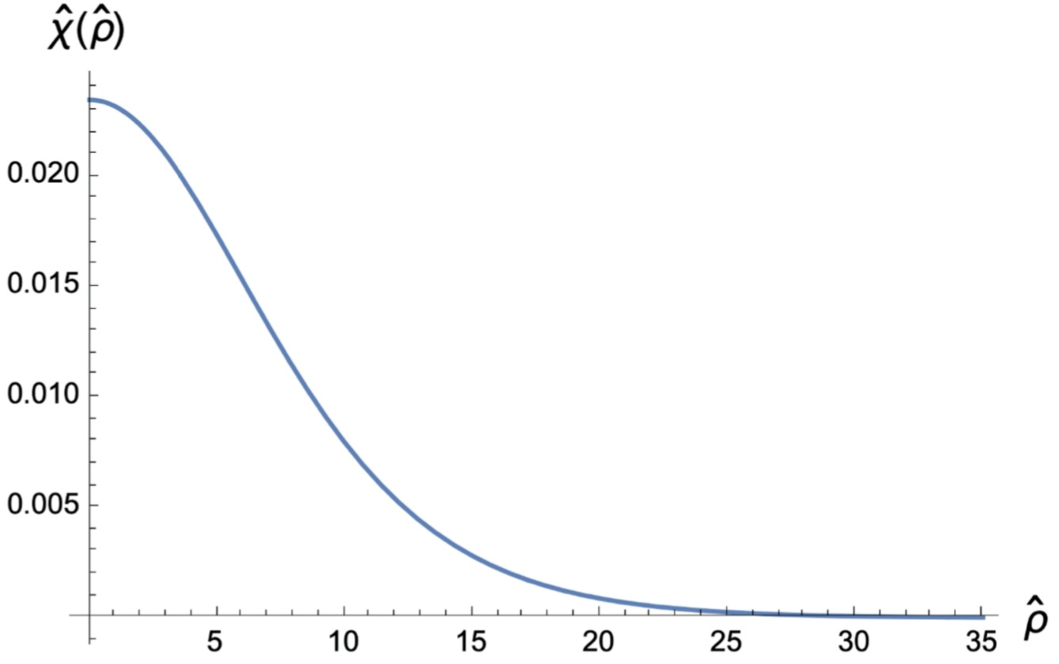
$$\begin{aligned} 0 &= -\nabla^2\chi + \left(m_\infty^2 + \frac{\kappa}{\alpha'}\varphi\right)\chi, \\ 0 &= -2\nabla^2\varphi + \frac{\kappa}{\alpha'}|\chi|^2. \end{aligned}$$

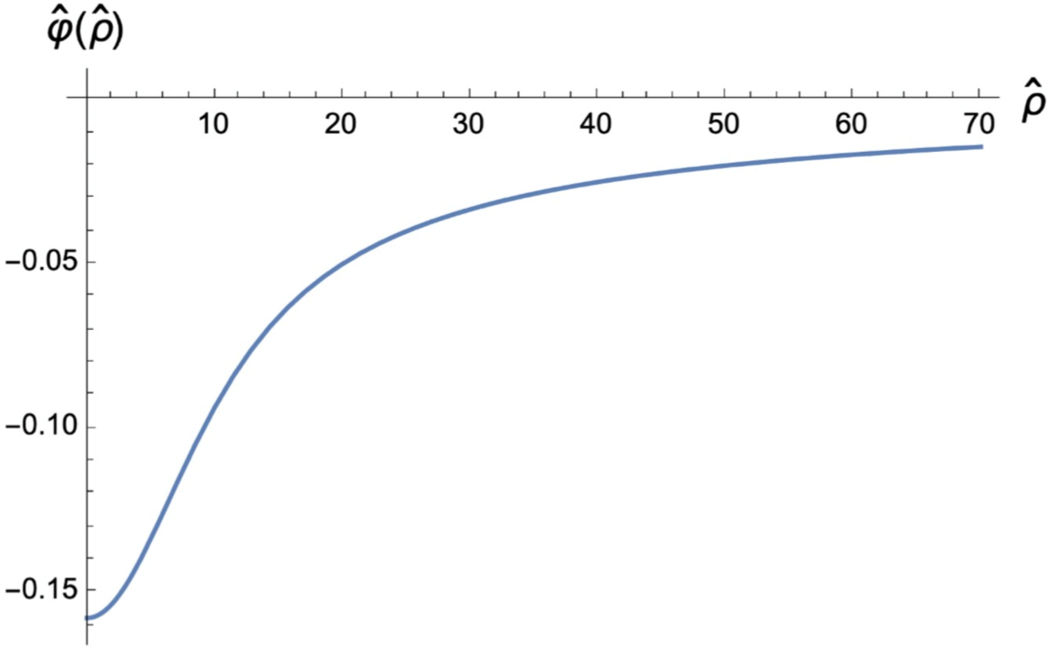
$$\beta - \beta_H \ll l_s$$

A string star solution by Horowitz and Polchinski

Spherical symmetric, normalizable solutions can be found in $D = 4, 5, 6$

For example, in $D = 4$





Circle is smaller in the center, but not vanishing!

A string star solution by Horowitz and Polchinski

It is a star solution of strings! We refer to it as the Horowitz-Polchinski solution.

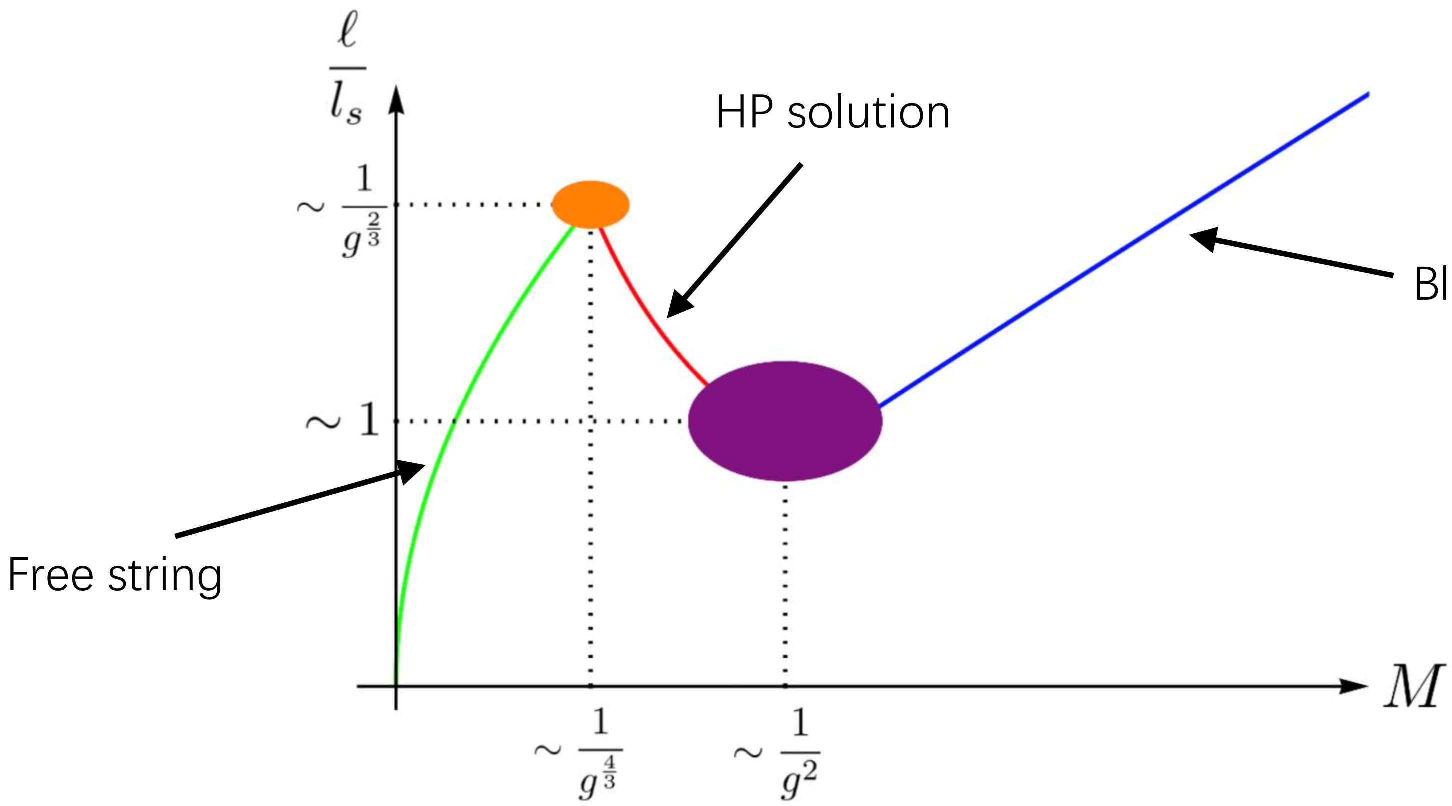
Even though it is constructed in Euclidean signature, where the winding mode is naturally defined ⋯

But we will see that its properties have nice interpretation in the Lorentzian signature.

Properties of the Horowitz-Polchinski solution

Size of the solution:

We can continue the metric (as well as the stress tensor) into the Lorentzian signature, and it gives us the size of a string star.



$D = 4$

Properties of the Horowitz-Polchinski solution

Thermodynamics of the solution:

For a Euclidean solution, we can adopt the Gibbons-Hawking procedure to compute its thermodynamic quantities.

In particular, $$S = (1 - \beta\partial_\beta)\log Z \sim \frac{\kappa}{\alpha'}\frac{\beta_H}{16\pi G_N}\int d^d x|\chi|^2$$

$$\longrightarrow \quad S = \beta_H M + \text{corrections}$$

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It matches the entropy of a highly excited string gas (with corrections).

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It matches the entropy of a highly excited string gas (with corrections).

Euclidean methods have been very successful in computing entropies in quantum gravity. It successfully produced the Page curve.

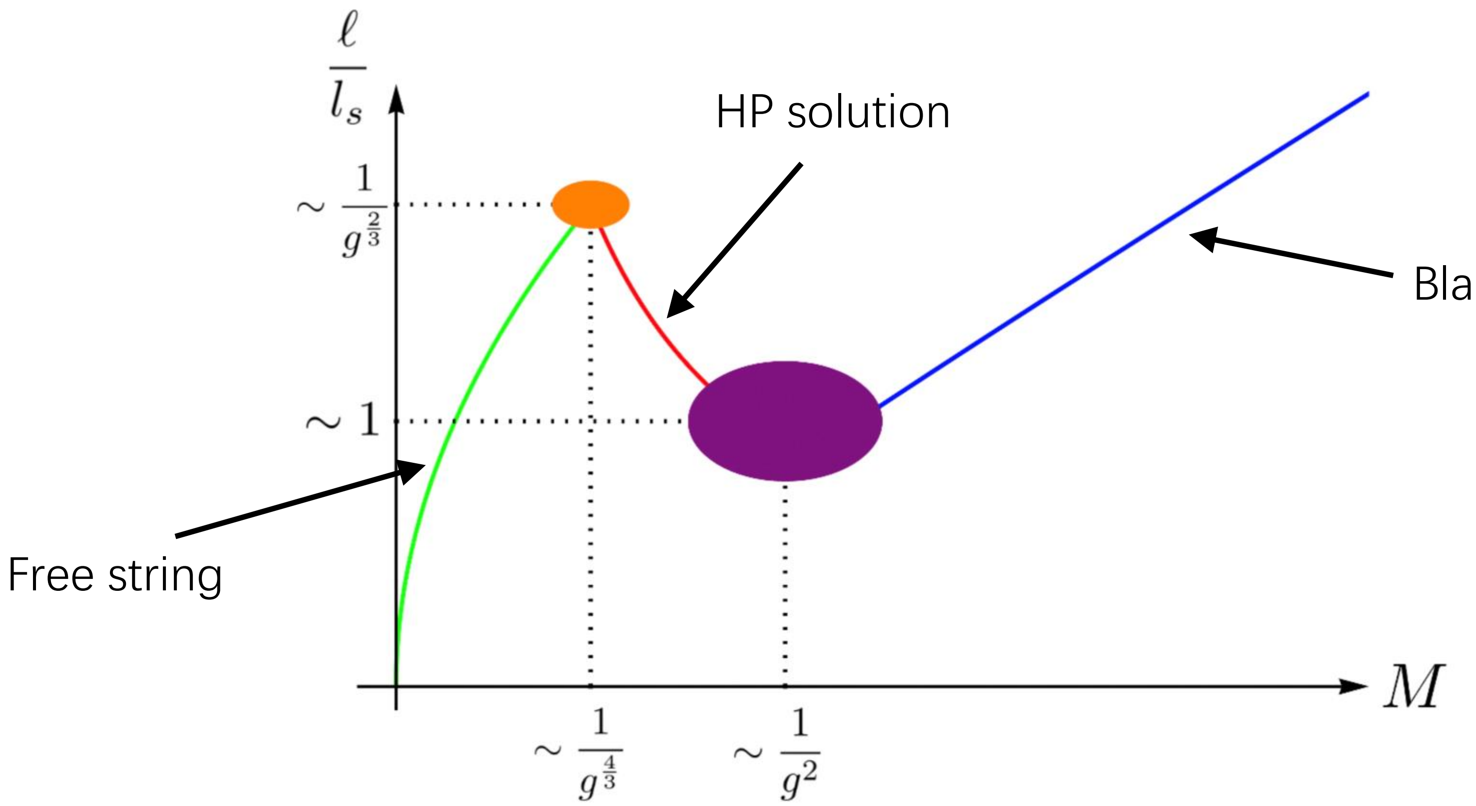
It is a nice example that we can compute the entropy through Euclidean method, while also knowing the **microstates** in the Lorentzian signature!

(we do not have it for the black hole)

Properties of the Horowitz-Polchinski solution

Transition to free string (orange blob):

- From the free string side, consider the Schrodinger equation for a small excitation in a gravitational potential, look at when a bound state forms.

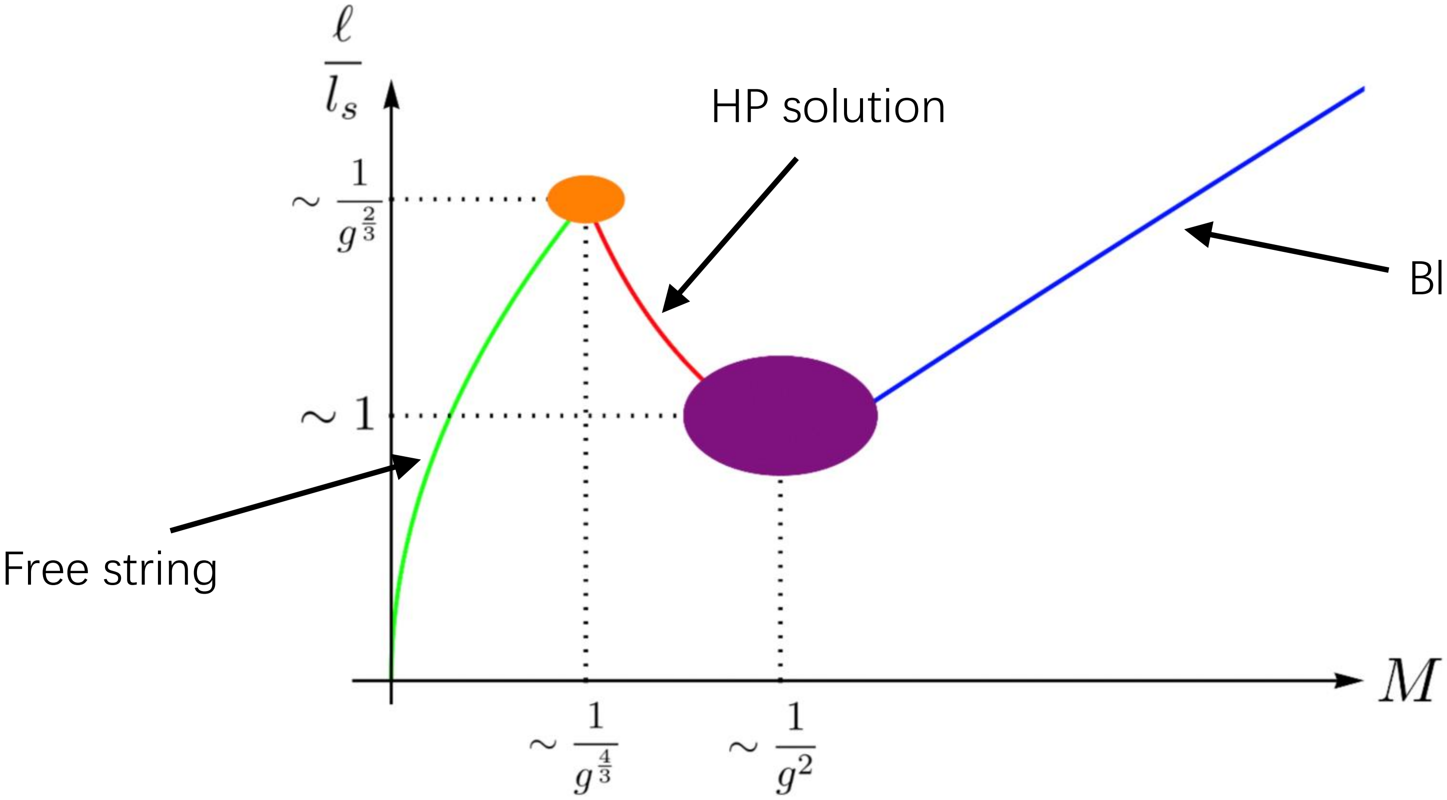


$$D = 4$$

Properties of the Horowitz-Polchinski solution

Transition to free string (orange blob):

- From the HP side, estimate when the quantum fluctuations around the classical solution become large.



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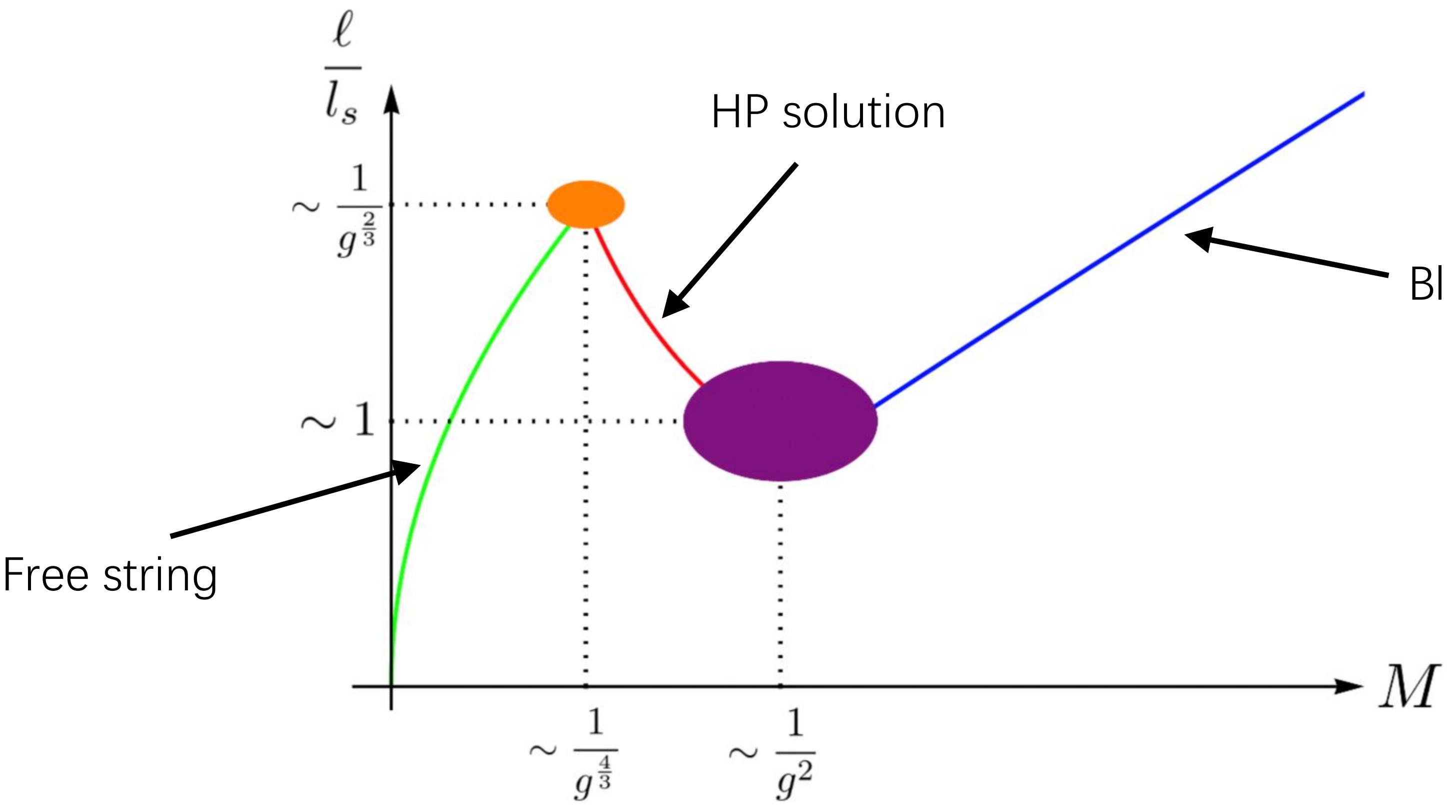
Properties of the Horowitz-Polchinski solution

The next part will be about the purple blob.



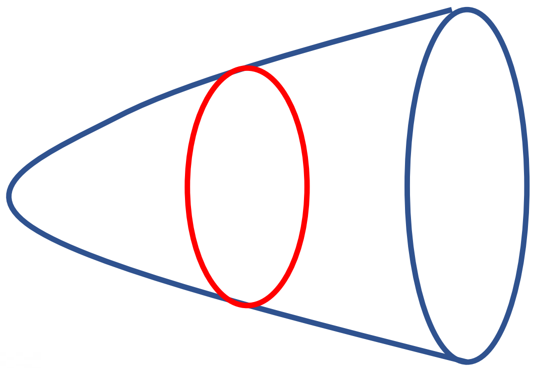
$$D = 4$$

Comparison between the HP solution and a black hole

- Both are classical solutions in string theory, and therefore they are both described by certain worldsheet CFTs.

- Validity: black hole $\beta \gg l_s$ HP solution $\beta - \beta_H \ll l_s$

- Both break the winding symmetry.
  (Landau paradigm does not apply)



- Both have an entropy at the classical level. $\sim 1/G_N$

- Black hole has a horizon (and an interior), while the HP solution does not.

Are the two CFTs smoothly connected?

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In short: we found different behavior in different string theories.

In Type II string theory: they cannot be smoothly connected as worldsheet CFTs.

In heterotic string theory: they are likely smoothly connected.

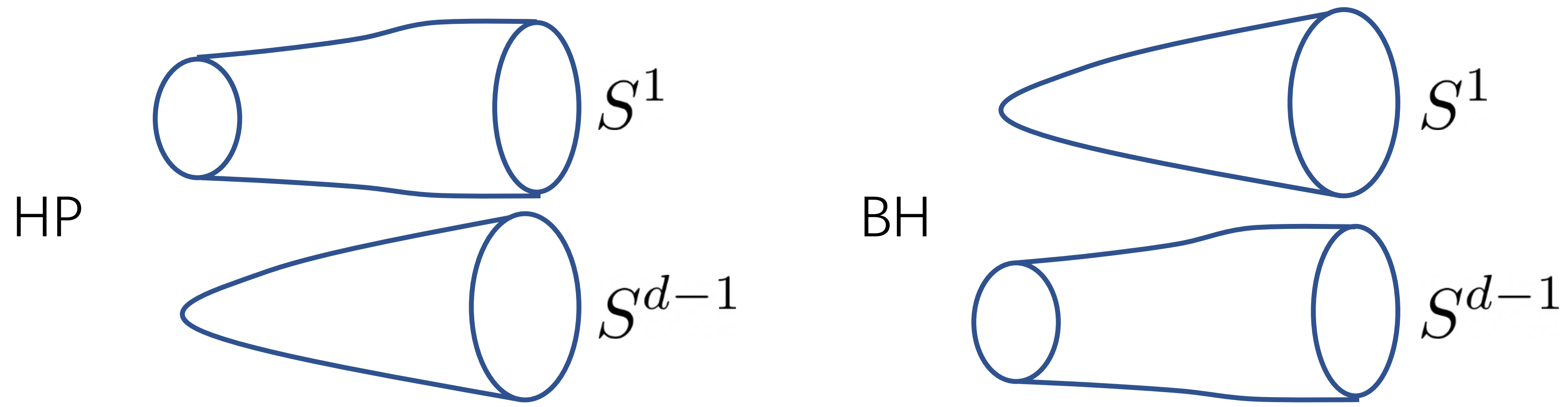
We did not consider the bosonic string since it has a tachyon.

Are the two CFTs smoothly connected?

Key idea: for supersymmetric sigma models, we can look at invariants that cannot vary smoothly with parameters.

Example: Witten index $\mathrm{Tr}_{\mathcal{H}}\,(-1)^F e^{-\hat{\beta} H}$ [Witten '82]

For sigma models with (1,1) worldsheet SUSY (Type II), it is equal to the Euler characteristic of the target space



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$$\mathrm{index}_{\mathrm{BH}} - \mathrm{index}_{\mathrm{HP}} = 2 \qquad \text{(Even D)}$$

So, there must be a singular point in the way.

For odd D, a small twist of the calculation will suggest the same conclusion.

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For sigma models with (0,1) worldsheet SUSY (heterotic), the index is equal to the index of the Dirac operator.

It vanishes for manifolds that are symmetric under parity transformation (both HP and BH).

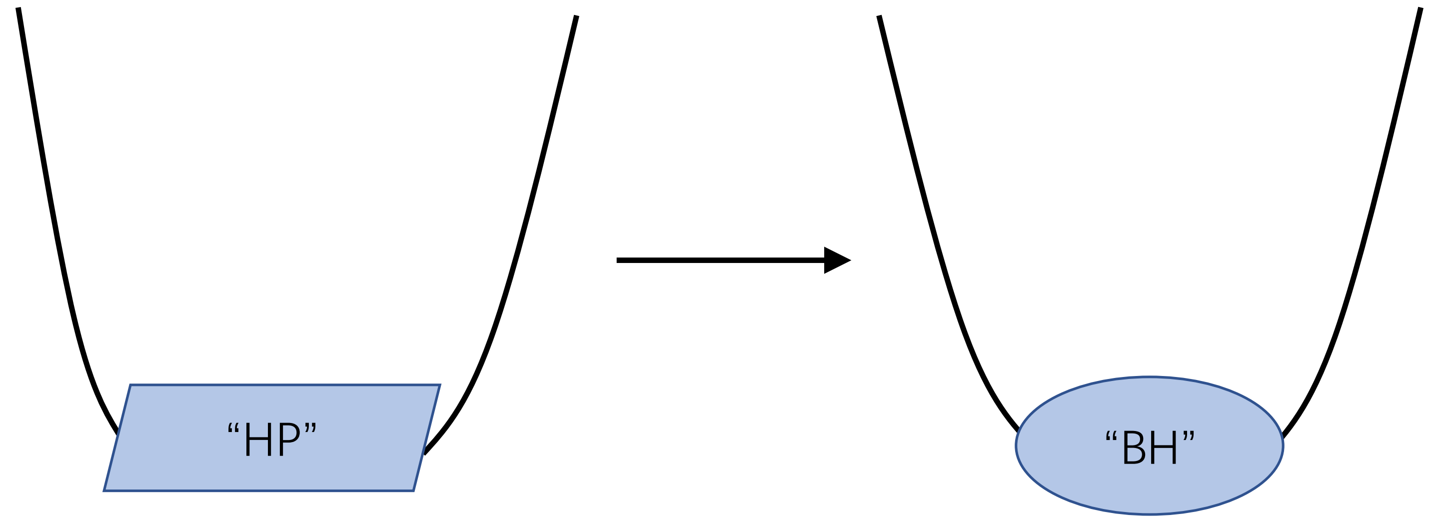
More generally, there are no known obstructions in the heterotic case.

An “off-shell” construction via linear sigma models

It is difficult to study the transition in the CFT directly. To gain more insight, it is useful to consider an “off-shell” problem.

Linear sigma model:

non-linear sigma model:



$\downarrow$ RG

CFT

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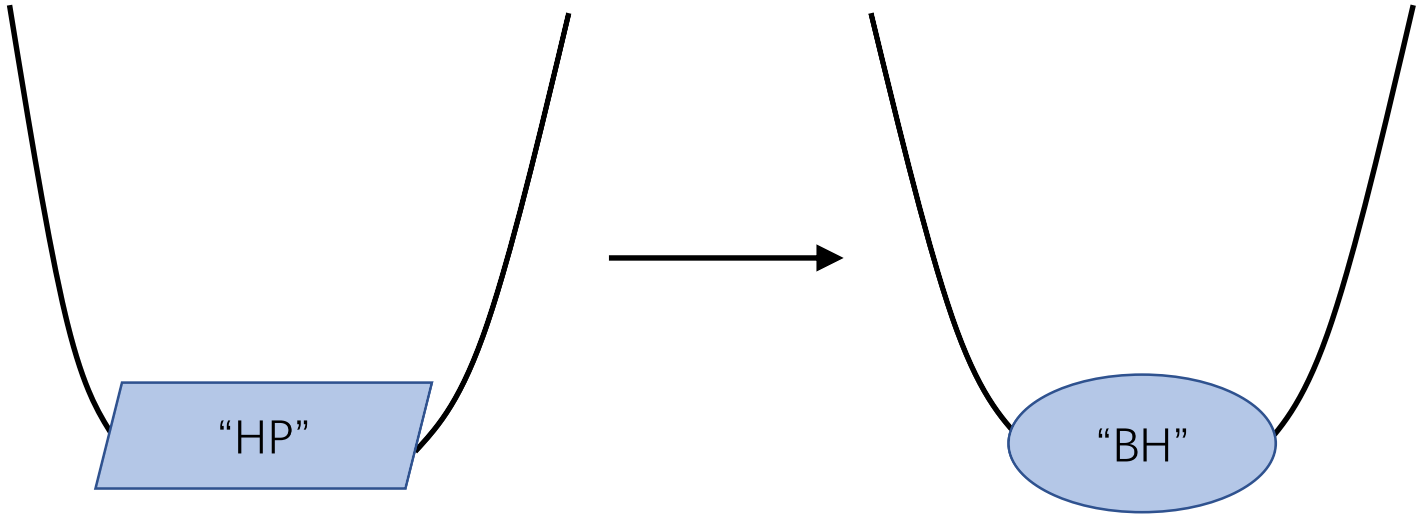

non-linear sigma model:

$\downarrow$ RG

CFT

- Note that we should be able to land on these CFTs under the RG (for example, the Schwarzschild solution is the unique solution with the right topologies).
- On the other hand, both the HP and the BH solutions have a negative mode, so we would need to fine tune some parameters in the UV.

[Headrick, Wiseman ’06, …]

A concrete construction

For the case with (1,1) SUSY (Type II), consider the following superpotential:

$$W = P((\vec{Y}^2 + a)(\vec{X}^2 - b) + c)$$

P, $\vec{X} = (X_1, X_2)$, $\vec{Y} = (Y_1, ..., Y_d)$ are scalar superfields.

Supersymmetric ground states satisfy:

$$(\vec{Y}^2 + a)(\vec{X}^2 - b) + c = 0, \qquad P\vec{Y}(\vec{X}^2 - b) = P(\vec{Y}^2 + a)\vec{X} = 0.$$

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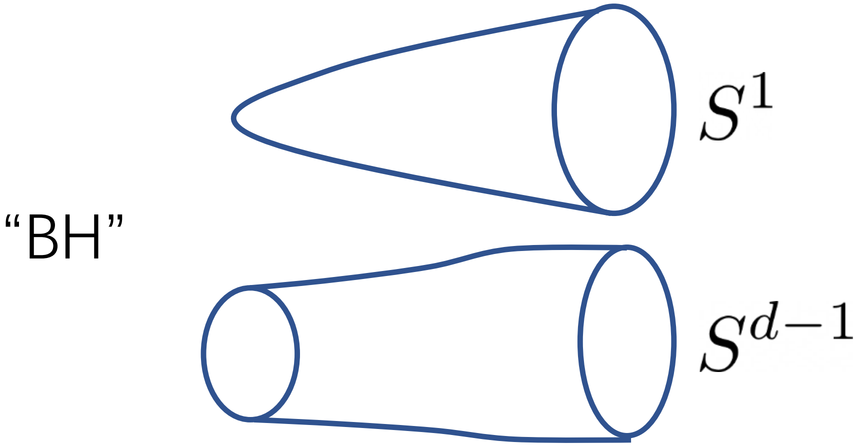
$c - ab > 0$



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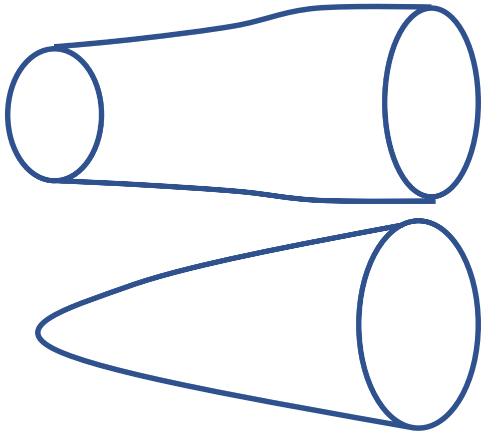
“HP”



S^1

S^{d-1}

$\langle P \rangle = 0$

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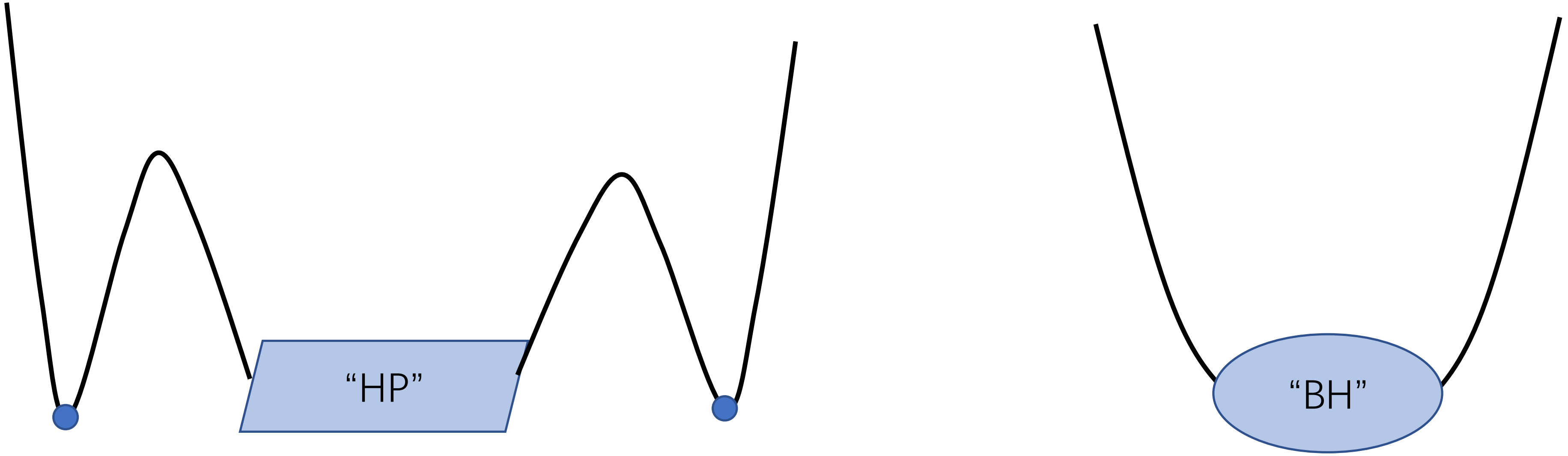
$c - ab = 0$ $\quad \langle X \rangle = \langle Y \rangle = 0,\ \langle P \rangle \neq 0$

$\langle P \rangle$

A concrete construction

Loop diagrams generate a potential for P, and give rise to massive vacua:

$$c - ab < 0 \qquad \langle X \rangle = \langle Y \rangle = 0,\ \langle P \rangle = f(a,b,c)$$



They compensate for the difference in the index found in the CFT.

Some comments

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- The massive vacua arise from the linear sigma model construction. Do they have an interpretation in the CFT (string theory)?

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- This specific construction does not behave too well under RG, but we proposed an improvement of it (updated in v2).
- The massive vacua arise from the linear sigma model construction. Do they have an interpretation in the CFT (string theory)?
- In the heterotic case, one can apply a similar construction. However, there is no analogue of massive vacua in that case, and the linear sigma model suggests a smooth transition.
- One cannot exclude the possibility that the transition is actually smooth in the type II theory but involves string loop effects that are not considered in our analysis. Of course, a phase transition is also possible.

Charged HP and black hole solutions

It is natural to wonder about charged HP solution and its relation to charged black holes.

Do we need to do all the analysis from scratch?

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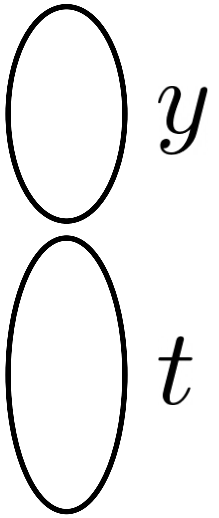
Do we need to do all the analysis from scratch?

No! We can start from the neutral ones and use the solution generating technique!

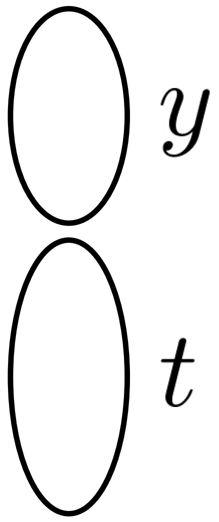
The original solution has non-zero energy in the time direction.

We can add an extra direction and generate momentum charge via boosting. A T-dual will give us the winding charge.



Charged HP and black hole solutions

The general procedure can be phrased in an abstract level that applies for CFTs, including both HP and black hole solutions.



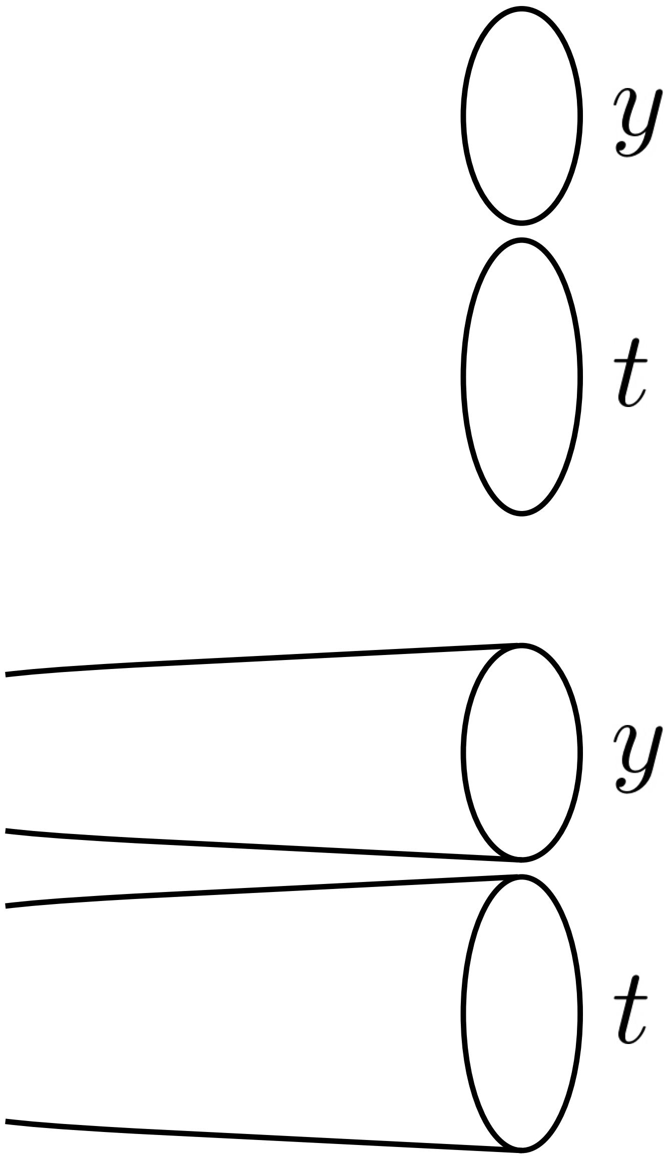
Charged HP and black hole solutions

The general procedure can be phrased in an abstract level that applies for CFTs, including both HP and black hole solutions.

The concrete transformation from the neutral to the charged solution, however, is expected to receive α' corrections in the interior of the spacetime.

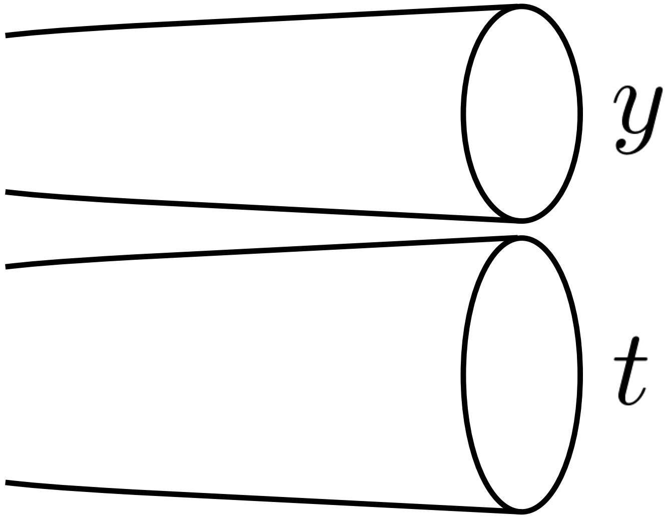
However, the **asymptotic form** of the transformation is good enough for our purpose! They have been discussed in the literature.

[Maharana, Schwarz ’92; …]



Charged HP and black hole solutions

The reason being that the action of a classical string theory solution is solely a boundary term, more specifically the Gibbons-Hawking-York term.



$$-I = \frac{1}{8\pi G_N}\int_{\partial\mathcal{M}} d^{D-1}y\, e^{-2\phi}\sqrt{h}(K - 2\partial_n\phi)$$

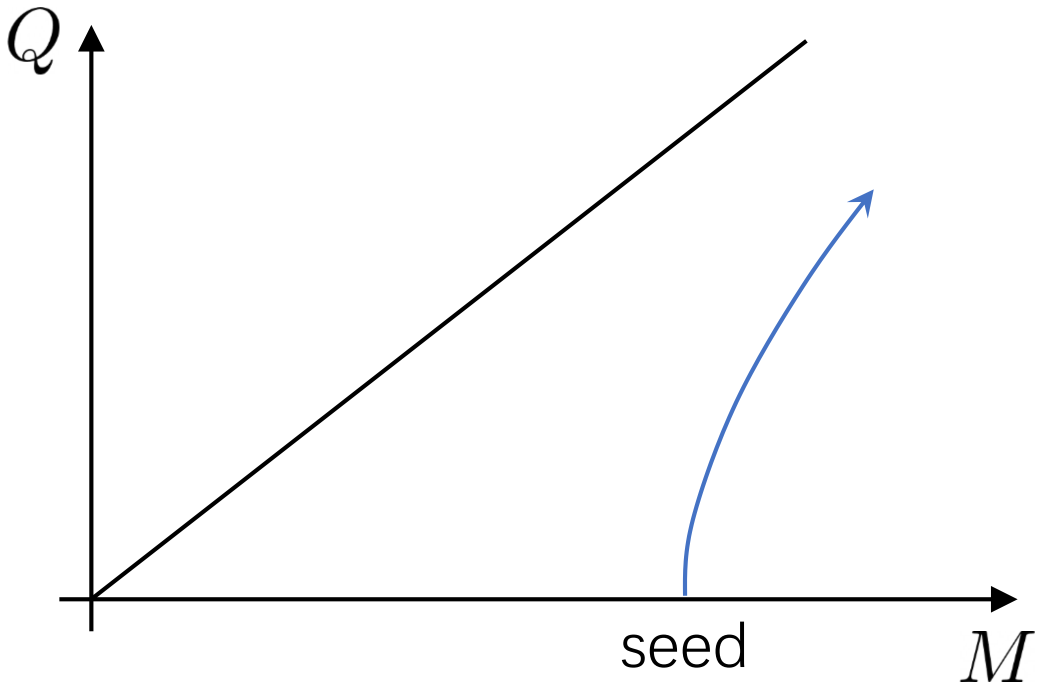
It can be simply evaluated knowing only the asymptotic form of the metric and dilaton:

$$\begin{aligned}\phi &\sim \frac{C_\phi}{\rho^{D-3}}\\ ds^2 &\sim e^{\frac{4\phi}{D-2}}\left[f dt^2 + \frac{d\rho^2}{f} + \rho^2 d\Omega^2_{D-2}\right], \qquad f \sim 1 - \frac{\mu}{\rho^{D-3}}\end{aligned}$$

Charged HP and black hole solutions

As a result, knowing the thermodynamics of a seed solution, one can work out all the thermodynamics of the charged solution generated from it.

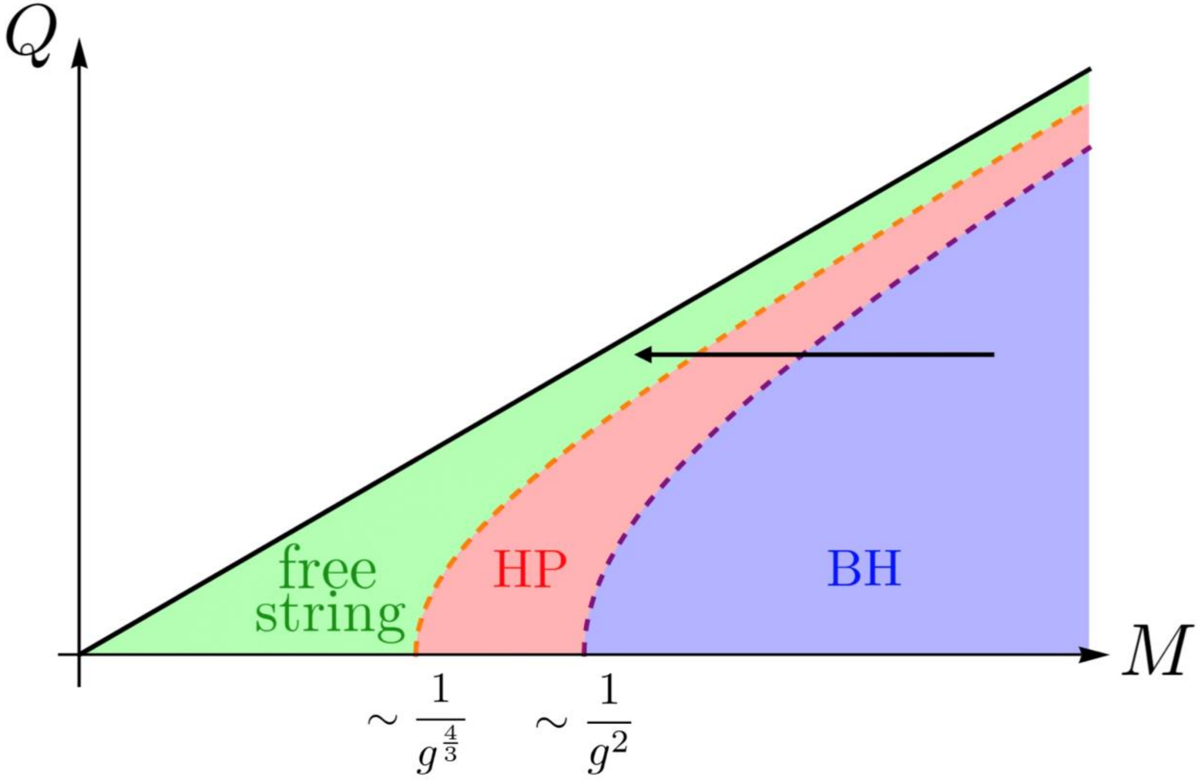
This result is exact in α' expansion.



The approach to extremality

This gives an update on the picture towards extremality.

For example, in D=4, we expect the following picture:



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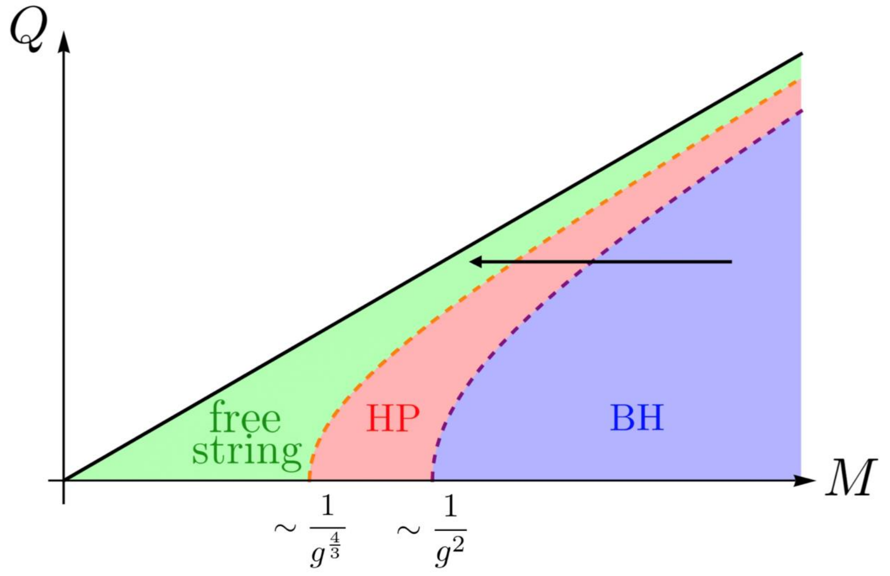
As we approach extremality, we transit from the black hole into a charged HP solution.

Note that the naïve extremal limit of this type of black hole is singular.

People have considered α' corrections to the singular solution and matched them with free string results. [Dabholkar '04; Sen '04]



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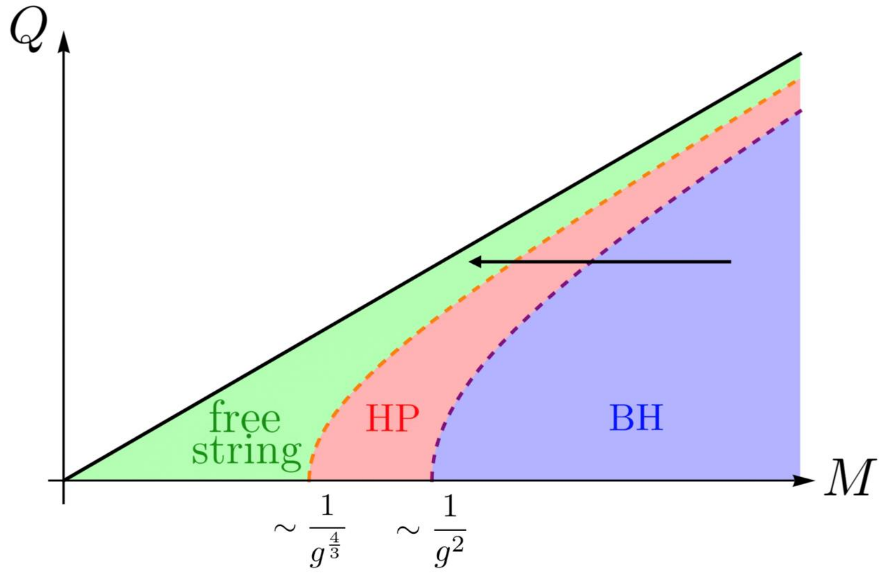
The charged HP solution naturally produces properties we expect for a string carrying charges (size, entropy, etc).

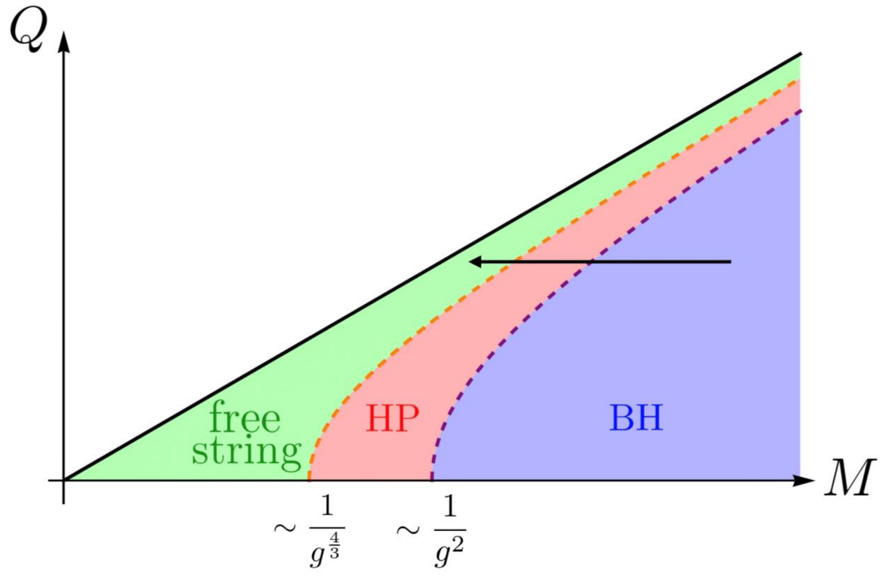


The approach to extremality

A side note: here we are talking specifically about perhaps the simplest charged black holes in string theory (carrying fundamental string momentum and winding charges).

There is a rich literature on the near extremal limit of other types of black hole.

It would be interesting to understand whether there is an analogue elsewhere.



Summary

- We revisited the Horowitz-Polchinski solution, which is a classical solution of a string star.
- It has a classical entropy, which has a clear Lorentzian interpretation.
- It cannot be smoothly connected to the black hole as classical solutions in type II string theory, while the opposite is likely in the heterotic string theory.
- We can generate charged solutions through the solution generating technique. In D=4, we go into the HP solution when approaching extremality.

Thank you!