Aspects of Gauge-Strings Duality

Carlos Nunez

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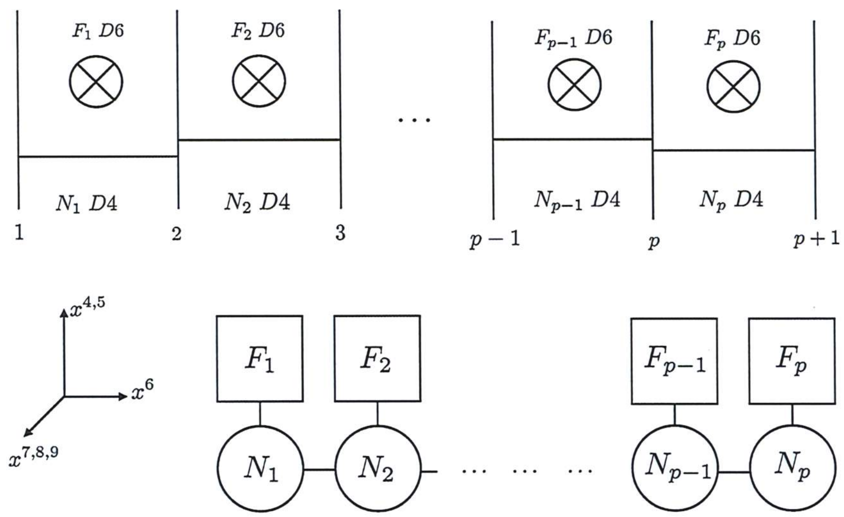
Outline

1. I will discuss work in the area of AdS/CFT. Focus on ideas and outcomes in favour of technical details.
2. Discuss the mapping between SCFTs and String backgrounds in diverse situations. The final goal: calculate correlation functions in the SCFTs, their deformation and RG-flows.
3. Today, I will focus on three examples: $N = 2$ SCFTs in four dimensions, $N = 4$ SCFTs in three dimensions, $N = 1$ SCFTs in five dimensions (last two **balanced**). Comments about RG-flow away from CFTs. The lessons discussed today are generic.
4. This talk is based on recent works with Mohammad Akhond and Andrea Legramandi. Very influenced by works published in the last few years with: Y. Lozano, N. Macpherson, A. Ramirez, S. Speziali, D. Thompson, D. Roychowdhuri, S. Zacarias.

A classification of a family of solutions of Type II or M-theory

Writing holographic duals to field theories: Conformal, SUSY, half BPS (8 SUSYs or 4 SUSYs). Dimension:1,2,3,4,5,6.
These theories have (at least) $SU(2)$ R-symmetry. They can have other flavour-like symmetries.

These QFTs are usually realised on Hanany-Witten set-ups with D_p, D_{p+2} and NS5 branes.



SCFTs in diverse dimensions (8+8 SUSY). An *incomplete* picture.

- d=6:Hanany-Zaffaroni, Brunner-Karch —D6-D8-NS5— Gaiotto, Tomasielo, Apruzzi, Fazzi, Rosa, Passias, Cremonesi.
- d=5:Aharony-Hanany-Kol —D5-D7-NS5– D'Hoker, Gutperle, Uhlemann, Trivella, Karch. Legramandi, Nunez
- d=4:Gaiotto —D4-D6-NS5— Gaiotto, Maldacena; Aharony, Berkooz, Berdichevsky; Stefanski, Reid-Edwards. Nunez, Speziali, Roychowdhuri, Zacarias.
- d=3: Gaiotto-Witten —D3-D5-NS5— D'Hoker, Estes, Gutperle; Assel, Bachas, Gomis. Akhond, Legramandi, Nunez.
- d=2:(0,4) SCFT –D2-D4-D6-D8-NS5. Lozano, Macpherson, Nunez, Ramirez. Couzens, Martelli, Schaffer-Nameki, Wong.
- d=1: $N=4$ SCQM– D0-D2-D4-D8-NS, Lozano, Nunez, Ramirez, Speziali.

Conformality, eight Poincare SUSYs. At least $SU(2)$ R-symmetry.

$$ds^2 \sim f_1 AdS_{d+1} + f_2 d\Omega_2 + f_3 d\Sigma_{7-d}(\vec{y}) \quad f_i(\vec{y}).$$

There are also NS B_2, Φ and RR fields respecting the isometries above.

Generically, what happens in all these cases is the following

We write the BPS equations. These are first order, nonlinear, coupled, partial differential equations.

Algebraic manipulations with these PDEs, allow us to write all the $f_i(\vec{y})$ in terms of a single function, that we refer to as 'potential function'.

Interestingly, the potential function satisfies a *linear* second order PDE (Laplace).

This Laplace equation needs of boundary conditions. Here is where the kinematic information about the SCFT enters, the good behaviour of the backgrounds is encoded, etc.

Examples: start with $N = 2$ four dimensional SCFTs.

Continue with $N = 4$ 3d SCFTs and $N = 1$ 5d SCFTs.

BPS equations $\rightsquigarrow f_i(V, \partial_\sigma V, \partial_\eta V, \dots) \rightsquigarrow$ linear PDE for $V(\sigma, \eta)$

$\downarrow$

Boundary and initial conditions

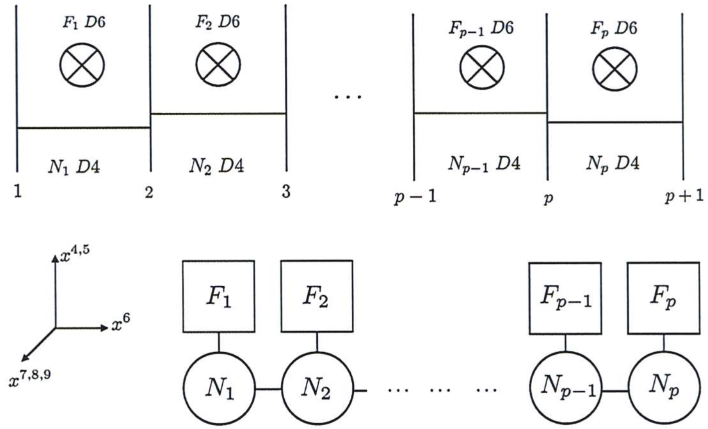
type II, M-theory background $\leftarrow$

$\uparrow$

QFT "data"

Consider 4d $\mathcal{N}=2$ SCFTs.

For its historical and conceptual value, we start with these 4d SCFTs. A quiver-like description and a Hanany-Witten set-up.



The global symmetries are $SO(2,4)\times SU(2)_R\times U(1)_r$. This will be reflected in the string dual containing $AdS_5\times S^2\times S^1$.

The beta function of these theories is $\beta\sim(2N_c-N_f)$. This implies

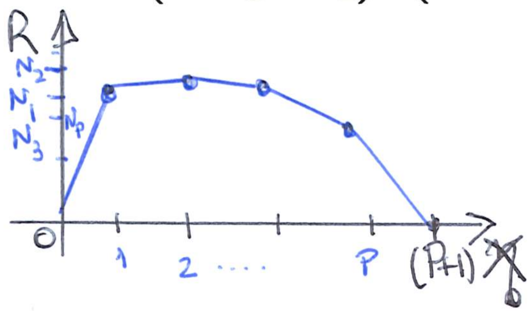
$$2N_1=N_2+F_1,\quad 2N_2=N_1+N_3+F_2,\ldots,2N_P=F_P+N_{P-1}.$$

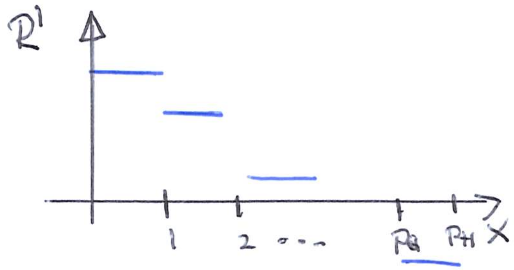
One can define a 'Rank-function' $R(x)$, that is convex polygonal. For the quiver pictured above, we have

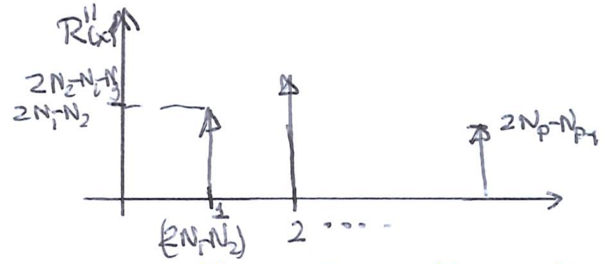
$$R(x)=\begin{cases} N_1 x & 0\leq x\leq 1\\ N_1+(N_2-N_1)(x-1) & 1\leq x\leq 2.\\ \ldots. & \\ N_k+(N_{k+1}-N_k)(x-k) & k\leq x\leq (k+1)\\ N_P-N_P(x-P) & P\leq x\leq P+1. \end{cases}$$

$$R'(x)=\begin{cases} N_1 & 0\leq x\leq 1\\ (N_2-N_1) & 1\leq x\leq 2.\\ \ldots. & \\ (N_{k+1}-N_k) & k\leq x\leq (k+1)\\ -N_P & P\leq x\leq P+1. \end{cases}$$

$$R''=(2N_1-N_2)\delta(x-1)+(2N_2-N_1-N_3)\delta(x-2)+\ldots.+(2N_P-N_{P-1})\delta(x-P).$$







Lin, Lunin and Maldacena wrote (2005) the Type IIA backgrounds ($\alpha' = g_s = 1$). Gaiotto-Maldacena (2010).

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi, \xi) + f_4 d\beta^2.$$

$$B_2 = f_5 d\Omega_2(\chi, \xi), \quad C_1 = f_6 d\beta, \; A_3 = f_7 d\beta \wedge d\Omega_2, \; e^{2\phi} = f_8.$$

The functions $f_i(\sigma, \eta)$ can be all written in terms of a function $V(\sigma, \eta)$ and its derivatives, $f_i \sim f_i(V, \partial_\sigma V, \partial_\eta V)$. For example

$$f_1(\sigma, \eta)^2 = \frac{2\dot{V} - \ddot{V}}{V''}, \quad f_2 = 2f_1 \frac{V''}{\dot{V}}, \ldots.$$

The function $V(\sigma,\eta)$ satisfies a Laplace-like equation with certain given boundary conditions to avoid nasty singularities,

$$\sigma^2\partial_\sigma^2 V + \sigma\partial_\sigma V + \sigma^2\partial_\eta^2 V = 0,$$
$$V(\sigma\to\infty,\eta)\to 0,$$
$$R(\eta) = \sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0}, \quad R(0) = R(P+1) = 0.$$

Importantly, the rank function $R(\eta)$ appears as an initial condition for the Laplace problem.

Given $R(\eta)$, one can write the solution for $V(\sigma,\eta)$ as a Fourier series

$$V(\sigma,\eta) = -\sum_{n=1}^{\infty}\frac{c_n}{w_n}K_0(w_n\sigma)\sin(w_n\eta), \qquad w_n = \frac{n\pi}{P+1}.$$
$$c_n = \frac{n\pi}{(P+1)^2}\int_{-(P+1)}^{P+1} R(\eta)\sin(w_n\eta)d\eta.$$

The backgrounds are trustable if the numbers P, N_k are large.

Using this, one can calculate the Page charges in correspondence with the number of D4,D6 and NS branes in the Hanany-Witten set-up.

$$Q_{NS5} = \frac{1}{4\pi^2}\int_{\eta,\Omega_2,\sigma\to\infty} H_3 = P+1,$$

$$Q_{D6} = \frac{1}{2\pi}\int_{\eta,\beta,\sigma=0} F_2 = R'(\eta=0) - R'(\eta=P+1),$$

$$Q_{D4} = \frac{1}{8\pi^3}\int_{\eta,\Omega_2,\beta,\sigma=0} F_4 - B_2\wedge F_2 = \int_0^{P+1} R(\eta)d\eta.$$

Linking numbers can be computed holographically. More interestingly, the central charge and Entanglement Entropy have expressions in terms of integrals of $R(\eta)$.

Let us see this central charge, in some detail.

The holographic central charge/Free Energy.

Roughly defined as a 'weighted version' of the volume of the internal space.

$$ds^2 = a(r,\vec{y})\left[-dt^2 + dx_d^2 + b(r)dr^2\right] + ds_{int}^2(r,\vec{y}), \quad \Phi(r,\vec{y}),$$

$$V_{int} = \int_{X_{int}} \sqrt{e^{-4\Phi}\det[g_{int}]a(r,\vec{y})^d}. \qquad H = V_{int}^2$$

$$c_{hol} \sim \frac{b(r)^{\frac{d}{2}}}{G_N}\frac{H^{\frac{2d+1}{2}}}{(H')^d}.$$

When computed for these backgrounds dual to 4d N=2 SCFTs,

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi,\xi) + f_4 d\beta^2, \; e^{2\phi} = f_8.$$

$$a = f_1(\sigma,\eta)r^2, \quad b = \frac{1}{r^4}, \; d = 3 \to V_{int} \to H \to$$

$$c_{hol} = \frac{1}{4}\int_0^P R(\eta)^2 d\eta.$$

$\boxed{AdS_5 \sim R^2 dx_{1,3}^2 + \frac{dR^2}{R^2}}$

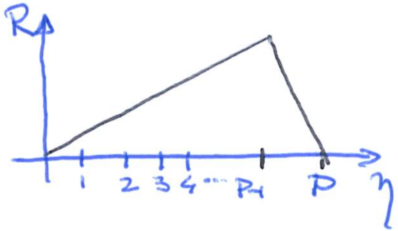
This can be compared with field theory results

There are field theory results

$$a=\frac{5n_v+n_h}{24},\qquad c=\frac{2n_v+n_h}{12}.$$

Generic expressions, for generic quivers. Focus on an example.

$$R(\eta)=\begin{cases} N\eta & ;(0,P-1)\\ N(P-1)(P-\eta) & ;(P-1,P)\end{cases}$$



We find

$$n_v=\sum_{k=1}^{P-1}N^2k^2-1,\quad n_h=\sum_{k=1}^{P-1}N^2k(k+1),$$

$$a=\frac{N^2P^3}{12}-\frac{5N^2P^2}{48}+\frac{P}{48}(N^2-10)+\frac{5}{24},$$

$$c=\frac{N^2P^3}{12}-\frac{N^2P^2}{12}-\frac{P}{6}+\frac{1}{6},$$

$$c_{hol}=\frac{1}{4}\int_0^P R(\eta)^2d\eta=\frac{N^2P^3}{12}-\frac{N^2P^2}{6}+\frac{PN^2}{12}.$$

Let us discuss the case of 3d *balanced* N=4 SCFTs.

N=4 SCFTs in d=3. $SO(2,3)\times SU(2)\times SU(2)$, 8 SUSYs. First discussed by D'Hoker, Estes and Gutperle (2007).

In the language of Akhond, Legramandi, Nunez. September 2021.

$$ds^2_{10,st} = f_1\Big[ds^2(\mathrm{AdS}_4) + f_2 d\Omega_{2,L} + f_3 d\Omega_{2,R} + f_4(d\sigma^2 + d\eta^2)\Big],$$

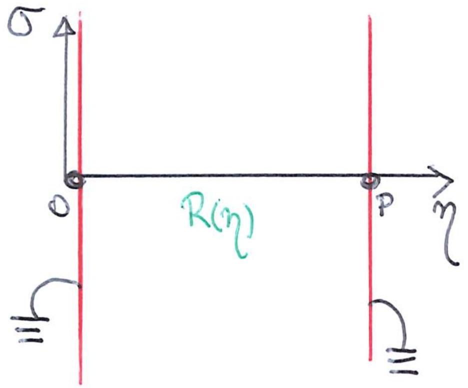
$$e^{-2\Phi} = f_5,\quad B_2 = f_6 \mathrm{Vol}\Omega_{2,L},\quad C_2 = f_7 \mathrm{Vol}\Omega_{2,R},\quad \tilde{C}_4 = f_8 \mathrm{Vol}(\mathrm{AdS}_4).$$

$$f_i(\sigma,\eta) = f_i(\hat{W}, \partial_\sigma \hat{W}, \partial_\eta \hat{W}, \ldots).$$

$$\partial_\sigma^2 \hat{W}(\sigma,\eta) + \partial_\eta^2 \hat{W}(\sigma,\eta) = \delta(\sigma)\mathcal{R}(\eta),$$

$$\hat{W}(\sigma,\eta=0) = 0, \qquad \hat{W}(\sigma,\eta=P) = 0,$$

$$\partial_\sigma \hat{W}(\sigma=0^+,\eta) - \partial_\sigma \hat{W}(\sigma=0^-,\eta) = -\mathcal{R}(\eta).$$



$$\begin{cases} \hat{W}(\sigma,\eta) = \sum_{k=1}^{\infty} a_k \left(\frac{P}{k\pi}\right)\cdot \sin\left(\frac{k\pi}{P}\eta\right)\cdot e^{-\frac{k\pi}{P}|\sigma|} \\ R(\eta) = \sum_{k=1}^{\infty} 2a_k \sin\left(\frac{k\pi}{P}\eta\right) \end{cases}$$

Quantising the Page charges.

$$N_{NS5} = \frac{1}{4\pi^2}\int_{\Sigma_3} H_3 = P$$

$$N_{D3} = \frac{1}{(2\pi)^4}\int_{\Sigma_5} \hat{F}_5 = \mathcal{R}(\eta) - (\eta - \Delta)\mathcal{R}'(\eta),$$

$$N_{D5} = \frac{1}{4\pi^2}\int_{\hat{\Sigma}_3} F_3 = \mathcal{R}'(\eta_i) - \mathcal{R}'(\eta_f).$$

This forces us to choose

$$\mathcal{R}(\eta) = \begin{cases} N_1\eta & 0 \leq \eta \leq 1 \\ N_l + (N_{l+1} - N_l)(\eta - l) & l \leq \eta \leq l+1, \quad l := 1, ..., P-2 \\ N_{P-1}(P - \eta) & (P-1) \leq \eta \leq P. \end{cases}$$

					SO(3)			SO(3)		
	t	x_1	x_2	y_1	y_2	y_3	z_1	z_2	z_3	η
NS5	—	—	—	·	·	·	—	—	—	·
D5	—	—	—	—	—	—	·	·	·	·
D3	—	—	—	·	·	·	·	·	·	—

A very similar calculation for the holographic central charge

$$c_{hol} = \frac{\mathcal{N}}{32\pi^6}, \quad \mathcal{N} = -16\pi^6 \int d\sigma d\eta \; \sigma(\partial_\eta^2 \hat{W})(\partial^3_{\sigma\eta\eta} \hat{W}).$$

$$c_{hol} = \frac{\pi}{8} \sum_{k=1}^{\infty} k a_k^2. \quad \text{(used Fourier expansion of } \hat{W} \text{)}$$

Generically, one gets an expression for c_{hol} in terms of $Li_3(N_i, F_i, P)$. Specified for,

$$R(\eta) = \begin{cases} N\eta & ; (0, P-1) \\ (P-1)N(P-\eta) & ; (P-1, P) \end{cases}$$

$$a_k = \frac{NP^2}{\pi^2 k^2} \sin\left(\frac{k\pi(P-1)}{P}\right),$$

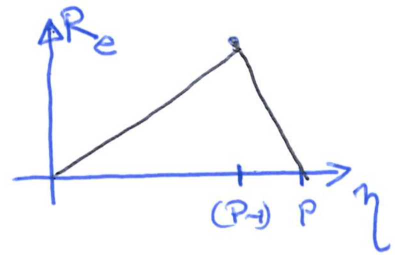
$$c_{hol} = \frac{N^2 P^4}{32\pi^3}\left[2\zeta(3) - 2\text{Re Li}_3(e^{\frac{2\pi i(P-1)}{P}})\right] \sim \frac{N^2 P^2}{8\pi} \log P.$$

[See also Cocca-Uhlemann 2021 Matrix model]

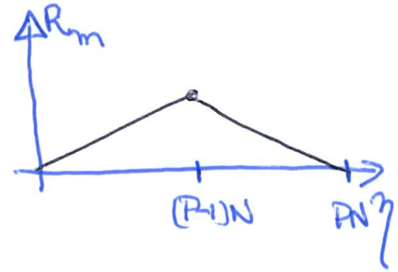
A nice thing, but non-generic!

Consider two **triangular** rank functions,

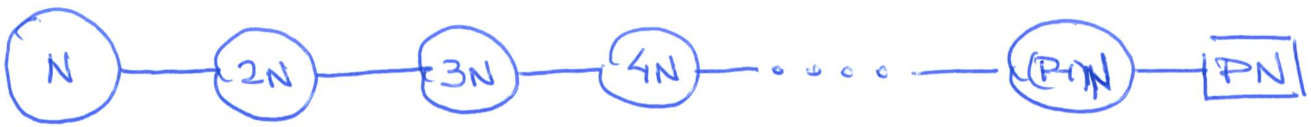
$$\mathcal{R}_e(\eta) = \begin{cases} N\eta & \eta \in [0, P-1] \\ (P-1)N(P-\eta) & \eta \in [P-1, P] \end{cases}$$



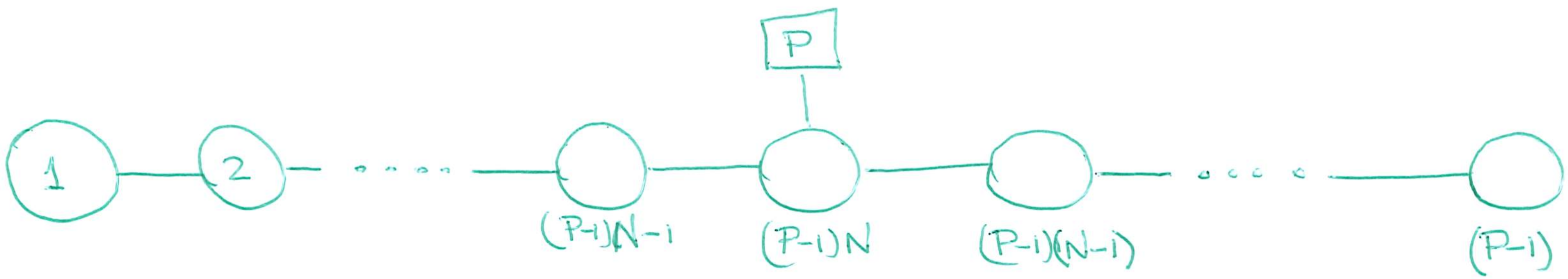
$$\mathcal{R}_m(\eta) = \begin{cases} \eta & \eta \in [0, (P-1)N] \\ (P-1)(NP-\eta) & \eta \in [(P-1)N, NP] \end{cases}$$



These correspond to the quivers



Quiver-e



Quiver-m

Calculate interesting numbers, using the first Rank function

$$N^e_{NS}=P,\ N^e_{D5}=PN,\ N^e_{D3}=\frac{NP(P-1)}{2},\ \hat{L}_{NS,i}=1,\ L_{D5,j}=N,$$
$$n^e_v=\frac{N^2P(P-1)(2P-1)}{6},\ n^e_h=\frac{N^2(P^3-P)}{3},$$
$$dim\mathcal{M}^e_H=n^e_h-n^e_v=\frac{N^2(P^2-P)}{2}.$$

For the second quiver/Rank function

$$N^m_{NS}=PN,\ N^m_{D5}=P,\ N^m_{D3}=\frac{N^2(P^2-P)}{2},\ \hat{L}_{NS,i}=N,\ L_{D5,j}=1,$$
$$n^m_v=\frac{N(P^2-P)(1+2N^2(P-1))}{6},\ n^m_h=\frac{N(P^2-P)(2+N^2(P-1)}{3}$$
$$dim\mathcal{M}^e_H=n^m_h-n^m_v=\frac{N(P^2-P)}{2}.$$

$$a^e_k=a^m_k \quad \text{and} \quad c^e_{hol}=c^m_{hol}.$$

What is going on?

Whilst this was discussed in an example, it works for any pair of triangular Rank functions, related by

$$\eta_e \leftrightarrow \frac{N_{NS5}}{N_{D5}}\hat{\eta}_m, \quad [a,b]_e \leftrightarrow \left[\frac{N_{D5}}{N_{NS5}}a, \frac{N_{D5}}{N_{NS5}}b\right]_m.$$

In general, this transformation maps

$$\begin{array}{ccc}
N^e_{NS5} & \longleftrightarrow & N^m_{D5} \\
N^e_{D5} & \longleftrightarrow & N_{NS5} \\
N^e_{D3} & \longleftrightarrow & \dim \mathcal{M}^m_H \\
\dim \mathcal{M}^e_H & \longleftrightarrow & N^m_{D3} \\
\hat{L}^e_{NS} & \longleftrightarrow & L^m_{D5} \\
L^e_{D5} & \longleftrightarrow & \hat{L}^m_{NS5} \\
a^e_k \ ; c^e_{hol} & \longleftrightarrow & a^m_k \ ; c^m_{hol}
\end{array}$$

This behaves as mirror symmetry. The non-generic character here is due to the fact that both initial and final quivers must be balanced!

Balanced 5d N=1 SCFTs. $SO(2,5)\times SU(2)$, 8 SUSYs.

This was initially studied in a series of works by D'Hoker, Gutperle, Uhlemann, Karch (2016). In the language of Legramandi Nunez, April 2021.

$$ds^2_{10} = f_1\Big[ds^2(\mathrm{AdS}_6) + f_2 ds^2(S^2) + f_3(d\sigma^2 + d\eta^2)\Big],$$

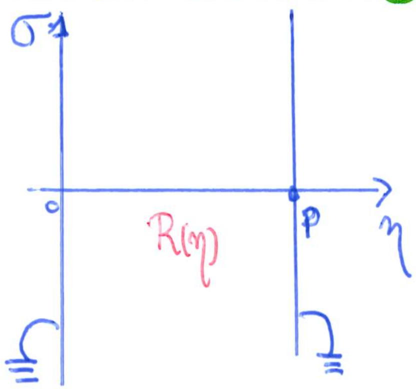
$$B_2 = f_4\mathrm{Vol}(S^2),\quad C_2 = f_5\mathrm{Vol}(S^2),\quad e^{-2\Phi} = f_6,\quad C_0 = f_7,$$

$$f_i(\sigma,\eta) = f_i(\hat{V},\partial_\sigma\hat{V},\partial_\eta\hat{V},....),\quad \partial^2_\sigma\hat{V} + \partial^2_\eta\hat{V} = \delta(\sigma)\mathcal{R}(\eta).$$

$$\hat{V}(\sigma\to\pm\infty,\eta) = 0,\qquad \hat{V}(\sigma,\eta=0) = \hat{V}(\sigma,\eta=P) = 0.$$

$$\lim_{\epsilon\to 0}\Big(\partial_\sigma\hat{V}(\sigma=+\epsilon,\eta) - \partial_\sigma\hat{V}(\sigma=-\epsilon,\eta)\Big) = \mathcal{R}(\eta).$$

Quantised Page charges, relates $\mathcal{R}(\eta)$ and a balanced quiver.



$\leftrightsquigarrow$

$$\begin{cases} \hat{V}(\sigma,\eta) = \sum_{k=1}^{\infty} a_k \sin\left(\frac{k\pi}{P}\eta\right) e^{-\frac{k\pi}{P}|\sigma|} \\ R(\eta) = \sum_{k=1}^{\infty}\left(\frac{2\pi k}{P}\right)\cdot a_k \sin\left(\frac{k\pi}{P}\eta\right). \end{cases}$$

Along the lines of the previous discussion.

The quantisation of Page charges forces the Rank function to be piecewise continuous and linear.

$$\mathcal{R}(\eta) = \begin{cases} N_1 \eta & 0 \leq \eta \leq 1 \\ N_k + (N_{k+1} - N_k)(\eta - k) & k \leq \eta \leq k+1, \quad k = 1, ..., P-2 \\ N_{P-1}(P - \eta) & (P-1) \leq \eta \leq P. \end{cases}$$

$$Q_{NS5} = P$$
$$Q_{D7}[k, k+1] = \mathcal{R}''(k) = (2N_k - N_{k+1} - N_{k-1}),$$
$$Q_{D5}[k, k+1] = \mathcal{R}(\eta) - \mathcal{R}'(\eta)(\eta - k) = N_k$$
$$Q_{D7,total} = \int_0^P \mathcal{R}''(\eta) d\eta, \quad Q_{D5,total} = \int_0^P \mathcal{R}\, d\eta.$$

Follow similar line as described above, compute $\hat{V}$ in Fourier series, central charge, etc.

For a generic rank function, like the one in the previous page, one can calculate $\hat{V}$ and c_{hol}.

$$\hat{V} = \frac{P^2}{2\pi^3} \sum_{s=1}^{P-1} N_{Fs} \mathrm{Re} \left(\mathrm{Li}_3(e^{-\frac{\pi(|\sigma|+i\eta-is)}{P}}) - \mathrm{Li}_3(e^{-\frac{\pi(|\sigma|+i\eta+is)}{P}}) \right).$$

$$c_{hol} = -\frac{P^4}{4\pi^{10}} \sum_{l=1}^{P-1} \sum_{s=1}^{P-1} N_{Fl} N_{Fs} \mathrm{Re} \left(\mathrm{Li}_5(e^{i\frac{\pi(l+s)}{P}}) - \mathrm{Li}_5(e^{i\frac{\pi(l-s)}{P}}) \right).$$

$$c_{hol} = \frac{1}{2\pi^4} \sum_{k=1}^{\infty} k a_k^2.$$

[See also
Uhlemann 2020
Matrix model
Santilli (2020)]

For the quiver we used as main example along this talk

$$\mathcal{R}(\eta) = \begin{cases} N\eta & 0 \le \eta \le (P-1) \\ N(P-1)(P-\eta) & (P-1) \le \eta \le P. \end{cases}$$

We find, for this quiver

$$a_k = (-1)^{k+1}\frac{NP^3}{k^3\pi^3}\sin\left(\frac{k\pi}{P}\right).$$

$$\hat{V} = \frac{NP^3}{2\pi^3}\mathrm{Re}\left(\mathrm{Li}_3(-e^{-\frac{\pi}{P}(|\sigma|+i+i\eta)}) - \mathrm{Li}_3(-e^{-\frac{\pi}{P}(|\sigma|-i+i\eta)})\right),$$

$$c_{hol} = \frac{N^2P^6}{8\pi^{10}}\left(2\zeta(5) - \mathrm{Li}_5(e^{\frac{2\pi i}{P}}) - \mathrm{Li}_5(e^{-\frac{2\pi i}{P}})\right) \sim \frac{N^2P^4}{2\pi^8}\zeta(3)$$

To close this, let me compare the results we got for the same quiver in the three dimensionalities considered

$$c_{hol}^{(3d)} \sim \frac{P^2N^2}{4\pi}\log P, \quad c_{hol}^{(4d)} \sim \frac{P^3N^2}{12}, \quad c_{hol}^{(5d)} \sim \frac{P^4N^2}{2\pi^8}\zeta(3).$$

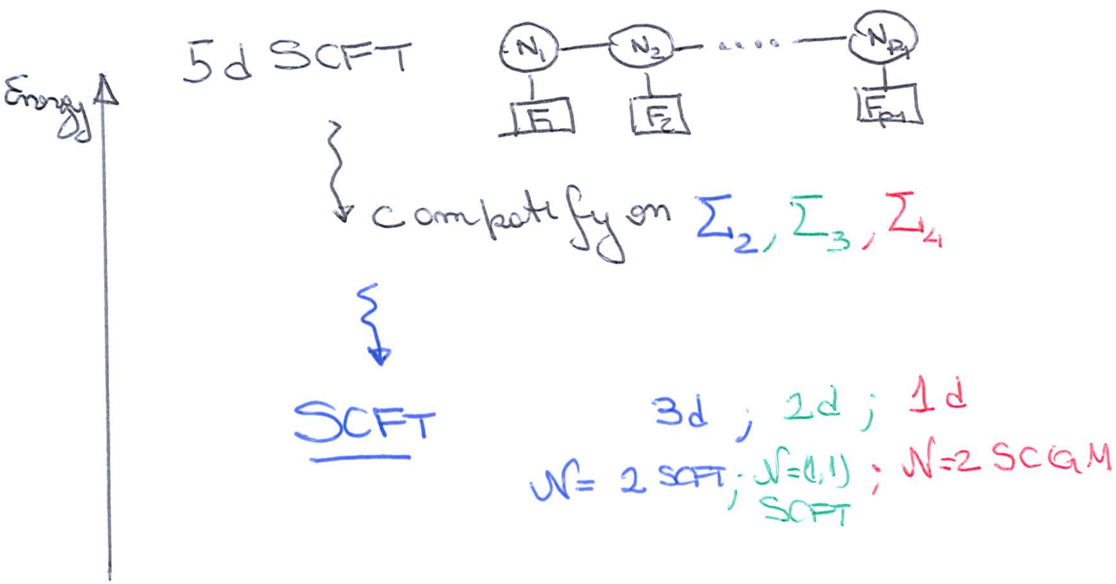
Let me leave this here and discuss some applications and predictions rather than tests!

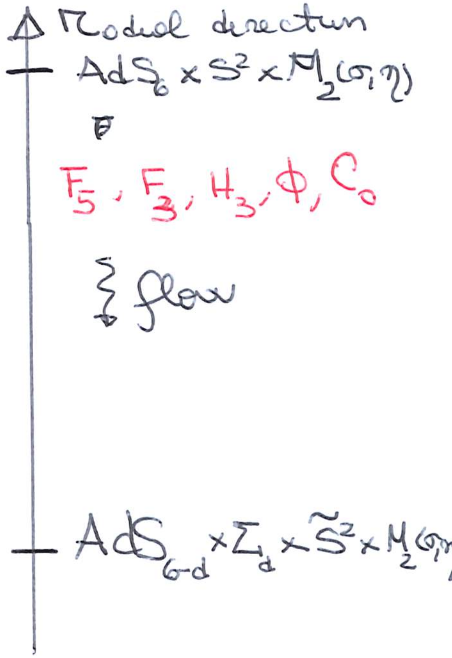
Let us discuss some applications and predictions

Consider RG-flows away from the 5d N=1 SCFTs.

We can compactify the 5d-SCFTs, deformed by relevant operators, leading at low energies to a lower dimensional QFT. Under certain circumstances, these are strongly coupled CFT_d, with $d = 1, 2, 3$.

This is captured by backgrounds constructed by Legramandi and myself (September 2021). QFT version by Sacchi, Sela, Zafrir (2021)





For the case SCFT5 flowing to SCFT3

$$ds^2_{st} = f_1\Big[e^{2f(r)}(-dt^2 + dy_1^2 + dy_2^2 + dr^2) + e^{2h(r)}\frac{(dz^2+dx^2)}{z^2} +$$
$$f_2 ds^2(\tilde{S}^2) + f_3(d\sigma^2 + d\eta^2)\Big], \quad f_i = f_i(\sigma, \eta, X(r))$$

$$B_2 = f_4 \mathrm{Vol}(\tilde{S}^2) + \frac{2}{9}\eta\frac{\cos\theta}{z^2}dx\wedge dz, \qquad C_0 = f_7,$$

$$C_2 = f_5 \mathrm{Vol}(\tilde{S}^2) + 4\partial_\sigma(\sigma V)\frac{\cos\theta}{z^2}dx\wedge dz,, \qquad e^{-2\Phi} = f_6.$$

$$F_5 = (1+*)\frac{2e^{4f(r)-2h(r)}}{3X^2}dr\wedge dt\wedge dy_1\wedge dy_2\wedge d\left(\cos\theta\sigma^2\partial_\sigma V\right),$$

$$ds^2(\tilde{S}^2) = d\theta^2 + \sin^2\theta(d\phi - A^3)^2, \quad \mathrm{Vol}(\tilde{S}^2) = \sin\theta d\theta\wedge\left(d\phi - A^3\right)$$

$UV \begin{cases} f(r) \sim h(r) \sim -\log r \\ \hookrightarrow AdS_6 \times S^2 \times M_2 \end{cases}$

$IR \begin{cases} f(r) \sim -\log r \\ h(r) = h_0 \end{cases}$

$AdS_4 \times \underbrace{H_2 \times \tilde{S}_2}\times M_2(\sigma,\eta)$

An infinite family of Type IIB backgrounds with AdS_4 factor, preserving four supercharges. Field theoretically, a twisted compactification of the $SCFT_5$ into three dimensions. There are BPS eqs. for $f(r), h(r), X(r)$.

One can define a central charge for this 'anisotropic flow'

The 'flow' central charge

We provided a formula for a monotonic quantity, detecting the UV AdS_6/CFT_5 and the AdS_4/CFT_3 fixed points. The result for this IR fixed point is

$$c_{IR} = \frac{\mathcal{N}_{AdS_4}}{G_N}\left(\frac{e^{f+h}}{f'}\right)^2, \qquad \mathcal{N}_{AdS_4} \sim \mathcal{N}_{AdS_6}\mathrm{Vol}[H_2].$$

In the IR fixed point $h \sim h_0, f \sim -\log r$. This matches nicely a generic picture advocated by Bobev and Crichigno (2015).

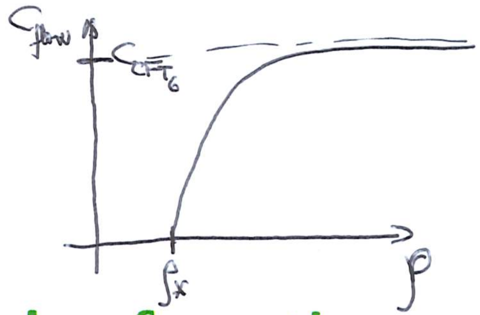
Similarly, construct holographic duals for and infinite family of backgrounds describing flows from 5d SCFT into 2d N=(1,1) SCFTs and 1d N=2 SCQM.

Consider SCFT5 flowing to a non-SUSY 4d theory

$$ds_{st}^2 = f_1\Big[e^{2\rho}\left(-d\tau^2 + d\vec{x}_3^2 + g(\rho)d\psi^2\right) + \frac{d\rho^2}{g(\rho)} + f_2 ds^2(S^2) +$$
$$f_3(d\sigma^2 + d\eta^2)\Big],$$
$$B_2 = f_4 \mathrm{Vol}(S^2), \quad C_2 = f_5 \mathrm{Vol}(S^2), \quad C_0 = f_7, \quad e^{-2\Phi} = f_6,$$
$$g(\rho) = 1 - e^{5(\rho_* - \rho)}.$$

The flow central charge, in this case reads

$$c_{flow} = \frac{\mathcal{N}_{AdS_6}}{16 G_N} \frac{\left(1 - e^{-5(\rho-\rho_*)}\right)^2}{\left(1 - \frac{3}{8} e^{-5(\rho-\rho_*)}\right)^4}.$$



Notice how the quantity divides into a part coming from the $SCFT_5$ and one coming from 'the flow". Also observed with Faedo and Rosen, in 2019.

Final words on applications

- It would be nice to understand how the Mirror symmetry described here applies in different dimensions. What are the characteristics of the 'mirror' backgrounds?
- It would be interesting to use the families of $AdS_{4,3,2}$ backgrounds to motivate more general classifications of backgrounds with $N = 2$, $N = (1, 1)$ and $N = 2$ SUSY.

[Example
Macpherson-Tomasiello
2021]

- The unbalanced quivers in this language is study in progress.
- In these classifications for backgrounds with AdS_D, for $D = 2, 3, 5, 7$, it is possible to find backgrounds on which the string sigma model is integrable (admits a Lax pair). The cases of AdS_4 and AdS_6 discussed today need to be worked out. Nicely, the integrable backgrounds are unique and always an integrable deformation of a solution known to be integrable.

Let me close with some conclusions

Some conclusions

The features discussed here for 4d $\mathcal{N}=2$ SCFTs, 3d $\mathcal{N}=4$ SCFTs and 5d $\mathcal{N}=2$ SCFTs, repeat in the cases of SUSY CFTs in 1d, 2d, 6d.

These systems can be thought in terms of D_p-D_{p+2}-NS_5 branes.

We study RG-flows away from these SCFTs, finding new SCFTs or a mass-gap behaviour. Interesting universality: some observables 'decouple' the flow from the CFT data.

It may be useful to unify the taxonomy of these backgrounds (and deformations). It is interesting to study how integrability and other observables behave under RG flows. It would be nice to extend these classifications to $\mathcal{N}=1$ cases.