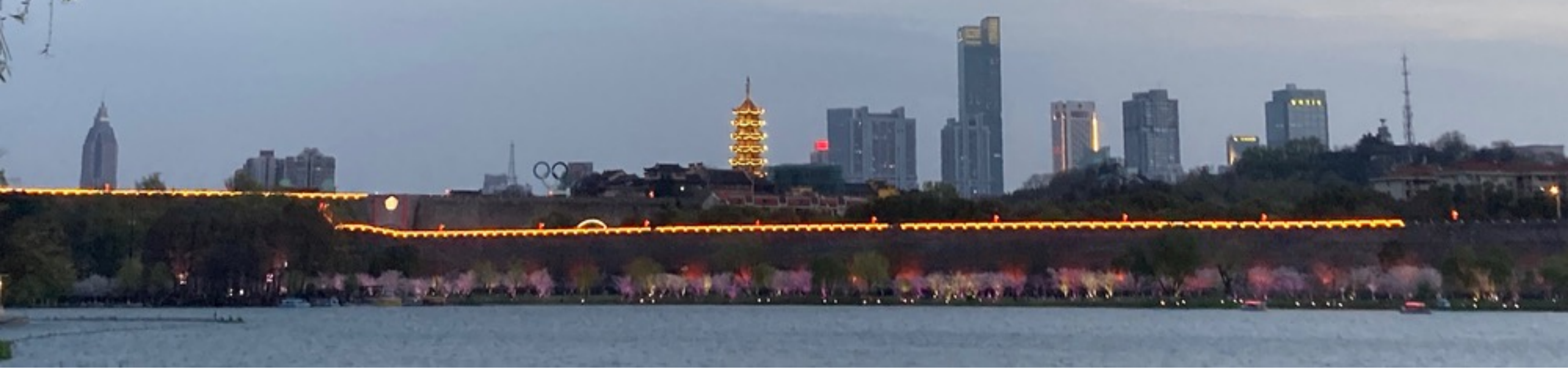

# Single particle operators in N=4 SYM and their correlation functions


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ICTP SAIFR South America Institute for Fundamental Research


*work with James Drummond, Paul Heslop, Hynek Paul, Francesco Sanfilippo, Michele Santangata, Alistar Steward in [2007.09395]*

# INTRODUCTION AND MOTIVATIONS

# $\mathcal{N}$=4 SYM

> most symmetric theory in 4d

> a lot of mathematical structure

> a lot of integrable structure

*arXiv:1012.3982 ; arXiv.1212.5605*

*many more reviews…*

scattering amplitude
correlators at weak coupling
one-loop dilatation operators

> quantum gravity

*SuperStrings on $AdS_5 \times S^5$ of radius L* $\longleftrightarrow$ *SU(N) $\mathcal{N}$=4 SYM in 4d*

$g_s = g^2_{YM}$ *and* $L^4 = \lambda \alpha^2$ *with* $\lambda = g_s N$

*Maldacena '90*

strong coupling through the bootstrap

SuperStrings on $AdS_5 \times S^5$ of radius $L$ $\longleftrightarrow$ $SU(N)$ $\mathcal{N}$=4 SYM in 4d

$g_s = g^2_{YM}$ and $L^4 = \lambda \alpha^2$ with $\lambda = g_s N$

Maldacena '90

Single particles (KK modes)

Half-BPS

$$\varphi_p \longleftrightarrow T_p + \frac{1}{N} \sum T_{p_1} T_{p_2} + \dots$$

Witten,
van Nieuwenhuizen et al.

$$N = \infty$$

hep-th/9806074 Minwalla, Seiberg Rangamani
hep-th/9908160 D'Hocker, Freedman, Mathur…Rastelli

hep-th/0003038 Arutyunov, Frolov

# THE PLAN

GIVE A DEFINITION OF SINGLE PARTICLE OPERATORS
DIRECTLY IN N=4 SYM,
THEREFORE BYPASSING THE GRAVITY COMPUTATION

AND STUDY ITS CORRELATORS

*FA, James Drummond, Paul Heslop, Hynek Paul in [1802.06889]*

*more recently -> [2007.09395]*

*let me remind you first about traces and notation...*

# Trace basis for half-BPS operators

$$T_p(x) = \mathrm{Tr}\Big[\underbrace{\phi(x)\ \ldots\ \phi(x)}_{p\ \text{times}}\Big] \qquad \phi(X,Y) = Y^R \phi_R(X)$$

consider all possible products of traces

$$T_{p_1 p_2 \ldots p_m}(x) = T_{p_1}(x) T_{p_2}(x) \ldots T_{p_m}(x)$$

$(\equiv T_{\underline{p}})$

# Wick contractions

$$\Big\langle {\phi_r}^s(X_1,Y_1) {\phi_t}^u(X_2,Y_2)\Big\rangle = \underbrace{\frac{Y_1 \cdot Y_2}{(X_1 - X_2)^2}}_{\equiv\ g_{12}} \times \begin{cases} \delta_r^u \delta_t^s & U(N) \\ \delta_r^u \delta_t^s - \frac{1}{N}\delta_r^s \delta_t^u & SU(N) \end{cases}$$

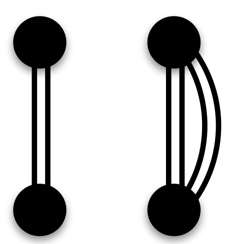
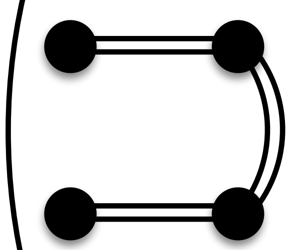
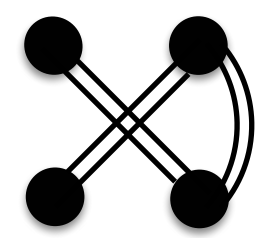
*there cannot be self-contractions on the same point

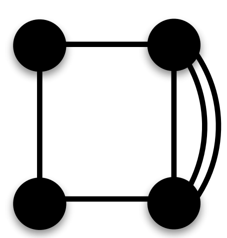
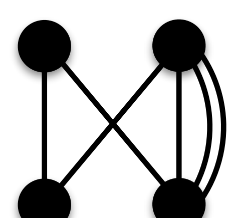
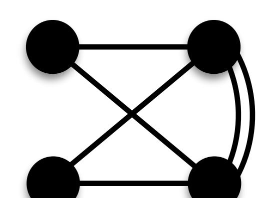
$g_{ij} =$ 

diagrammatic notation:
correlator = sum over prop. structures

for example….

$\langle T_2(x_1)T_2(x_2)T_4(x_3)T_4(x_4)\rangle =$

$\frac{8(N^2-1)^2(N^4-6N^2+18)}{N^2}$  $+ \frac{32(N^2-1)(2N^2-3)^2}{N^2}\Bigg($  $+$  $\Bigg) +$

$\frac{64(N^2-1)(N^4-6N^2+18)}{N^2}\Bigg($  $+$  $\Bigg) + \frac{192(N^2-1)(N^4-6N^2+18)}{N^2}$ 

## …beyond the planar limit

Trace basis is degenerate

$$T_p \quad ; \quad T_{p_1} T_{p_2}\Big|_{p_1+p_2=p} \quad ; \quad \dots$$

There are trace relations for $T_{p>N}$

The gravity picture is actually nicer

KK modes have a notion of orthogonality

Exclusion principle in $AdS_5 x S^5$: A graviton with large angular momentum pops into a D3 brane wrapping an $S^3$ -> it will have maximal radius! and cut-off with N

*J. McGreevy, L. Susskind and N. Toumbas hep-th/0003075*

Since the AdS/CFT is non perturbative we would like to understand which basis is the one dual to KK modes

There is a simple solution* !!

$$\textit{Single particles operator} \equiv \left\{ \mathcal{O} \ : \ \left\langle \mathcal{O}_p(x_1) T_{q_1,\ldots q_n}(x_2) \right\rangle = 0 \right\}$$

(in turn single particle are orthogonal to multiparticles)

for illustration:

$$\mathcal{O}_4 = T_4 - \frac{(2N^2-3)}{N(N^2+1)} T_{22} + \frac{10}{N^2+1} T_{211} - \frac{4}{N} T_{13} - \frac{5}{N(N^2+1)} T_{1111}$$

$$\mathcal{O}_5 = T_5 - \frac{5(N^2-2)}{N(N^2+5)} T_{32} + \frac{15(N^2-2)}{N^2(N^2+5)} T_{221} + \ldots$$

** Of course this agrees, in the large N limit, with the subleading corrections of [1806.09200]*

**Orthogonality was long appreciated in AdS3xS3 e.g. Marika Taylor hep-th/0003075*

BACK TO THE PLAN

## What I will tell about the SP basis:

We have three ways of writing the SP operators

- on the trace basis
- on the Schur basis
- on the eigenvalue basis

We know the 2pt function normalisation

We know that SP interpolate between T and giant gravitons

We can prove a multi-point orthogonality theorem

We understand ME, NME correlators, well

# SP operators, from scratch

partitions of p

$$\mathcal{O}_p(x) = \frac{1}{\mathcal{N}_p} \det \begin{pmatrix} \mathcal{C}_{\lambda_1|\lambda_1} & \mathcal{C}_{\lambda_2|\lambda_1} & \dots & \mathcal{C}_{p|\lambda_1} \\ \vdots & \vdots & \dots & \vdots \\ \mathcal{C}_{\lambda_1|\lambda_{P-1}} & \mathcal{C}_{\lambda_2|\lambda_{P-1}} & \dots & \mathcal{C}_{p|\lambda_{P-1}} \\ \hline T_{\lambda_1}(x) & T_{\lambda_2}(x) & \dots & T_p(x) \end{pmatrix}$$

$$\{\lambda_i = q_1 \dots q_n\}_{i=1,\dots P}$$

$$\lambda_P \equiv \{p\}$$

from GS orthogonalisation w.r.t. the metric induced by the 2pt function

$$\mathcal{N}_p = \det \left(\mathcal{C}_{\lambda_i\lambda_j}\right)_{1\leq i,j\leq P-1} \qquad ; \qquad \langle T_{\lambda_i}(x_1) T_{\lambda_j}(x_2)\rangle = g_{12}^p \, \mathcal{C}_{\lambda_i\lambda_j}$$

example:

minors from the above det

$$\mathcal{O}_4 = T_4 - \frac{(2N^2-3)}{N(N^2+1)} T_{22} + \frac{10}{N^2+1} T_{211} - \frac{4}{N} T_{13} - \frac{5}{N(N^2+1)} T_{1111}$$

# SP operators more concretely

*T. W. Brown [hep- th/0703202]*

*the dual of the trace basis*

$$\langle \xi_{\lambda_I}[\phi] \cdot T_{\lambda_J} \rangle = \delta_{IJ}$$

$$\mathcal{O}_p(x) = \sum_{\{q_1,\ldots,q_m\}\vdash p} C_{q_1,\ldots,q_m} T_{q_1,\ldots,q_m}(x)$$

$$C_{q_1,..q_m} = \frac{|[\sigma_{q_1..q_m}]|}{(p-1)!} \sum_{s\in\mathcal{P}(\{q_1,..,q_m\})} \frac{(-1)^{|s|+1}(N+1-p)_{p-\Sigma(s)}(N+p-\Sigma(s))_{\Sigma(s)}}{(N)_p-(N+1-p)_p}$$

…about the notation

power set, e.g.

$$\mathcal{P}(\{3,2,1\}) = \{\{\},\{1\},\{2\},\{3\},\{2,1\},\{3,1\},\{3,2\},\{3,2,1\}\}$$

given a subset “s” in the power set, then |s| is the cardinality and

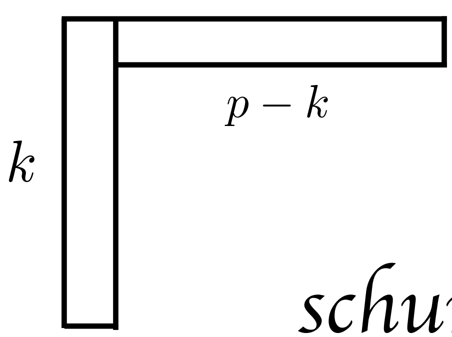
$\Sigma(s) = \sum_{s_i\in s} s_i$ finally $|[\sigma_{q_1..q_m}]|$ is the size of conj. class

# SP operators more concretely

$$\mathcal{O}_p = \sum_{k=1}^{p} d_k(p,N) E_{p,k}(z_i)$$

*eigenvalue basis*

$$d_k(p,N) = (-1)^k \frac{(p-1)_k}{(N-k+1)_k} \frac{p}{1-\frac{(N)_p}{(N-p+1)_p}}$$




*schur basis*

$$\mathcal{O}_p = \sum_{k=1}^{p} \tilde{d}_k(p,N) \chi_{R_k^p}[\phi]$$

$$\tilde{d}_k(p,N) = (-)^k \frac{(p-1)}{p+N} \frac{(N-k+p+1)_k}{(N-k+1)_k} \frac{p}{1-\frac{(N)_p}{(N-p+1)_p}}$$

---

…comparing examples

$$\mathcal{O}_3 = \tfrac{2}{N^2} T_{111} - \tfrac{3}{N} T_{21} + T_3$$

$$\mathcal{O}_3 = \tfrac{(N-2)(N-1)}{N^2} E_{3,1} - \tfrac{3(N-2)}{N^2} E_{3,2} + \tfrac{12}{N^2} E_{3,3}$$

$$\mathcal{O}_3 = \tfrac{(N-2)(N-1)}{N^2} \chi_{R_1^3} - \tfrac{(N-2)(N+2)}{N^2} \chi_{R_2^3} + \tfrac{(N+1)(N+2)}{N^2} \chi_{R_3^3}$$

# Immediate properties

1. SP operators in U(N) are SP operators in SU(N)

$$\Pi: \; O[\phi] \;\to\; \hat{O}[\phi] \equiv O[\hat{\phi}(\phi)] \qquad\qquad \hat{\phi}_r{}^s = \phi_r{}^s - \tfrac{1}{N}\delta_r^s \phi_t{}^t \;,$$

$$\mathcal{O}_p^{U(N)}[\phi] = \hat{\mathcal{O}}_p^{SU(N)}[\hat{\phi}] \qquad p \geq 2$$

2. two point function normalisation from dual basis construction

$$\langle \mathcal{O}_p(x_1)\mathcal{O}_p(x_2)\rangle = g_{12}^p \times p^2(p-1)\left[\frac{1}{(N-p+1)_{p-1}} - \frac{1}{(N+1)_{p-1}}\right]^{-1}$$

$$\frac{p}{N^{p-2}} \times \prod_{r=1}^{p-1}(N^2-r^2) \times \Big(1 + \frac{\#}{N^2} + \ldots$$

normalisation analytic in N and p, it vanishes when p>N

for p>N all operators are indeed multitrace! single trace not indp.

# SP interpolate btw Traces and D3 branes

if N is large, and p is small SP ~ single trace

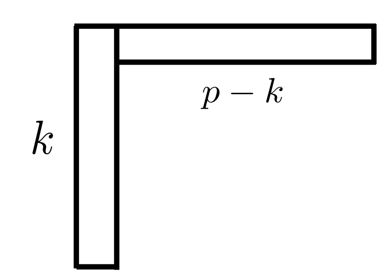
$$\mathcal{O}_p \;\to\; T_p \;+\; \text{“}\tfrac{1}{N}\text{”} \qquad\qquad p \ll N$$

if N is large, and p ~ N then SP is very different from T

in fact I just showed you that SP =0 for p>N

$$\mathcal{O}_p \;\to\; (-)^{p+1} p \left( \frac{1}{1+\frac{p}{N}} \right)^{N-p+1} E_{p,p} \quad ; \qquad p, N \gg 1 \text{ and } p-N \text{ fixed}$$

$$E_{p,p} \;=\; z_1 \ldots z_p + \ldots \;=\; \chi_{R_p^p}$$

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$\mathcal{O}_p = \sum_{k=1}^{p} d_k(p,N) E_{p,k}(z_i)$ there is an infinite sum but compared to the leading term the other are exponentially suppress by Gamma factors

# SP interpolate btw Traces and giant D3 branes

if N is large, and p is small SP ~ single trace

$$\mathcal{O}_p \;\rightarrow\; T_p \;+\; \text{“}\tfrac{1}{N}\text{”} \qquad\qquad p \ll N$$

if N is large, and p ~ N then SP is very different from T

in fact I just showed you that SP =0 for p>N !!

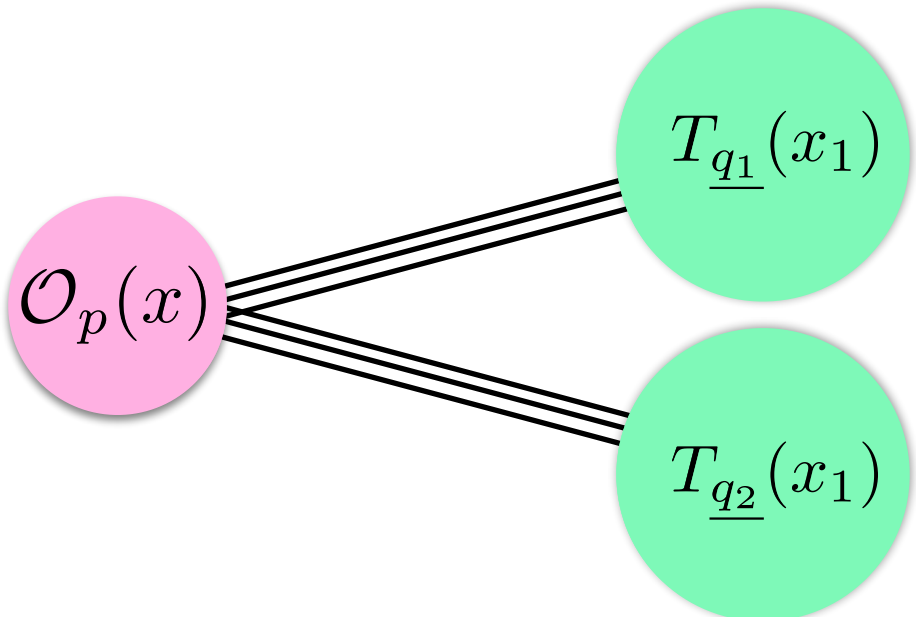
$$\mathcal{O}_p \;\rightarrow\; (-)^{p+1} p \left( \frac{1}{1+\frac{p}{N}} \right)^{N-p+1} E_{p,p} \quad ; \qquad p, N \gg 1 \text{ and } p-N \text{ fixed}$$

$$E_{p,p} \;=\; z_1 \ldots z_p + \ldots \;=\; \chi_{R_p^p}$$

The operator identified as the dual of the giant D3 brane

# MULTI-POINT ORTHOGONALITY

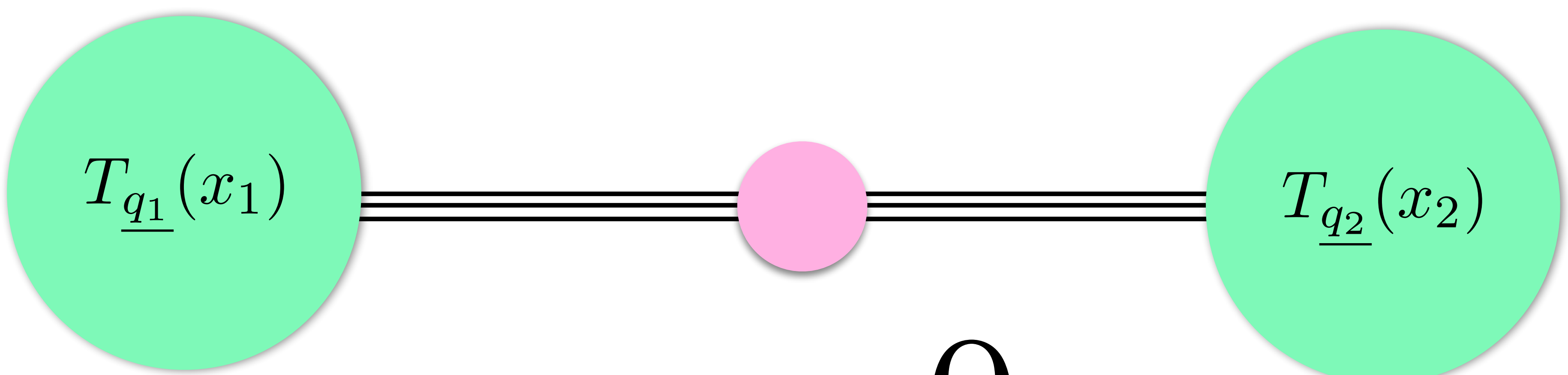
# Extremal Three-point functions vanish




$$p = q_1 + q_2$$

The color factor of this two point  function vanishes by definition/construction

by moving one multi-trace on a ficticious point we get



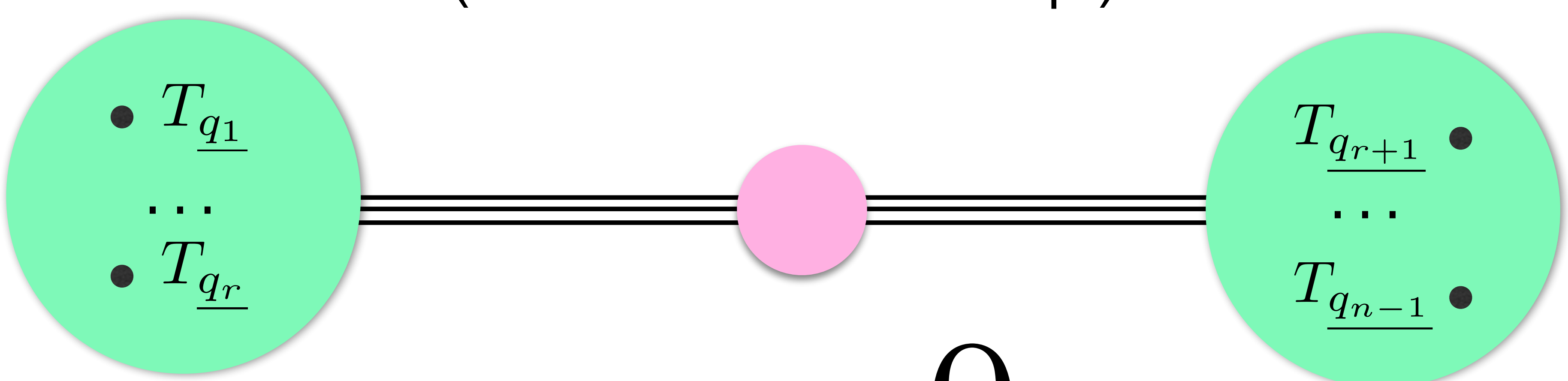

$$= 0$$

# Multi-point orthogonality

Theorem: Any propagator structure with a SP operator insertion that is connected to two sub-diagrams, themselves disconnected from each other, has vanishing color factor

$$\mathcal{F}_{p|\underline{q_1}\ldots\underline{q_{n-1}}}(x, x_1 \ldots x_{n-1}) = \langle \mathcal{O}_p(x) T_{\underline{q_1}}(x_1) \ldots T_{\underline{q_{n-1}}}(x_{n-1}) \rangle$$

(a dumbbell shap)




$$= 0$$

## … some intuition from combinatorics

$$\mathcal{F}_{p|\underline{q_1}\dots\underline{q_{n-1}}}(x, x_1 \dots x_{n-1}) = \langle \mathcal{O}_p(x) T_{\underline{q_1}}(x_1) \dots T_{\underline{q_{n-1}}}(x_{n-1}) \rangle$$

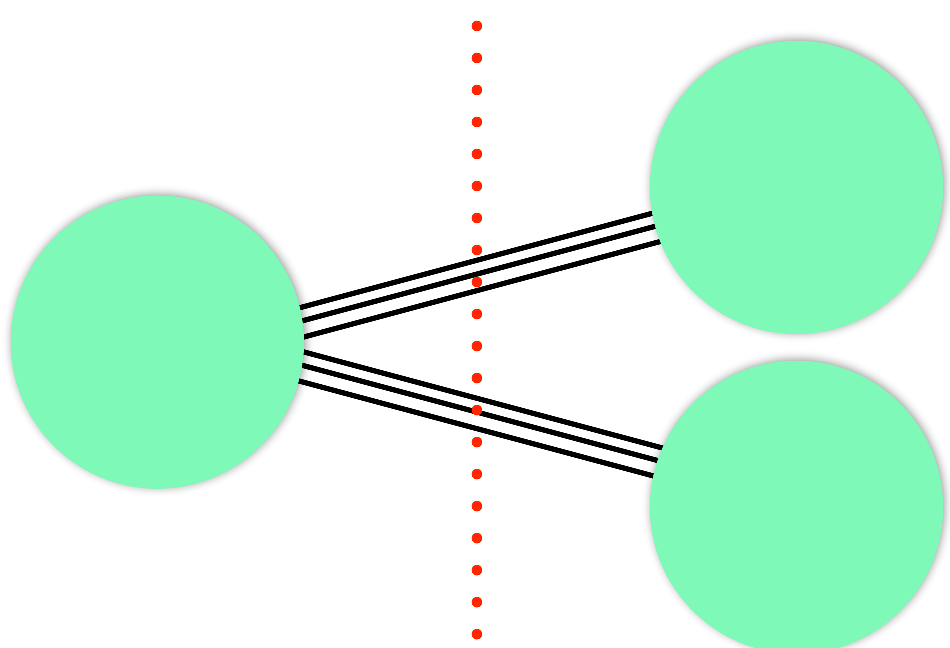
becomes the following determinant

$$\frac{1}{\mathcal{N}_p} \det \left( \begin{array}{cccc} \mathcal{C}_{\lambda_1|\lambda_1} & \mathcal{C}_{\lambda_2|\lambda_1} & \dots & \mathcal{C}_{p|\lambda_1} \\ \vdots & \vdots & \dots & \vdots \\ \mathcal{C}_{\lambda_1|\lambda_{P-1}} & \mathcal{C}_{\lambda_2|\lambda_{P-1}} & \dots & \mathcal{C}_{p|\lambda_{P-1}} \\ \hline \mathcal{C}_{\lambda_1|\underline{q_1}\dots\underline{q_{n-1}}} & \mathcal{C}_{\lambda_2|\underline{q_1}\dots\underline{q_{n-1}}} & \dots & \mathcal{C}_{p|\underline{q_1}\dots\underline{q_{n-1}}} \end{array} \right)$$

where

$$\mathcal{C}_{\lambda_i|\underline{q_1},\dots,\underline{q_{n-1}}} \simeq \langle T_{\lambda_i}(x) T_{\underline{q_1}}(x_1) \dots T_{\underline{q_{n-1}}}(x_{n-1}) \rangle_{\mathcal{F}}$$

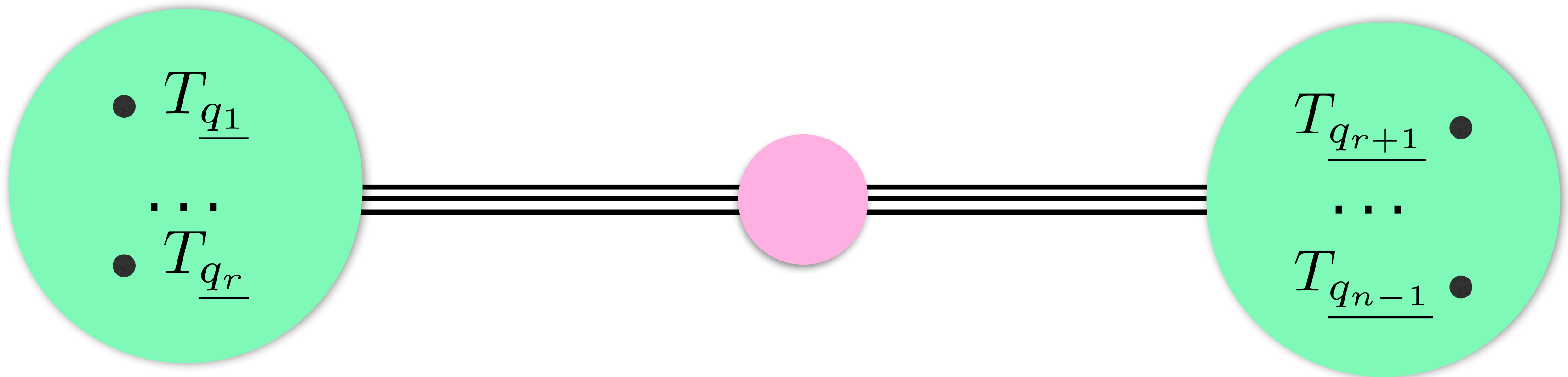
$$\mathcal{C}_{\lambda_i|\lambda_j} \simeq \langle T_{\lambda_i} T_{\lambda_j} \rangle$$



$\lambda_i$ are partitions of p

$$\lambda_P \equiv \{p\}$$

! Topology is a dumbbell !




here there can be also contractions between traces on the same blob inserted at different point, and still have a dumbell -> one parameter:

$$\frac{1}{2}\left(-p+\sum_{i=1}^{n-1} q_i\right) = k \geq 0$$

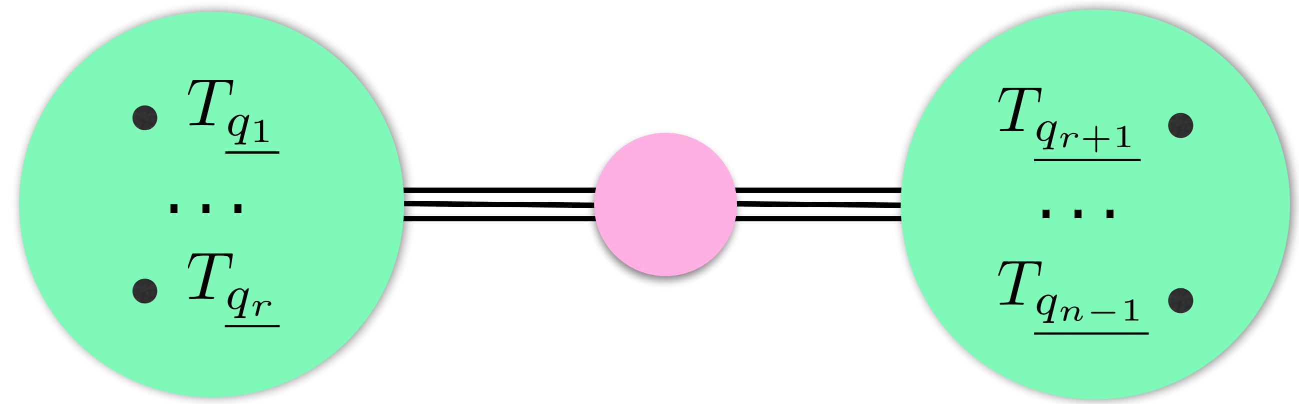
it will follow a classification of correlators

prove that only dumbbell

1. near extremal n-pt correlators (NE) $k \leq n-3$
2. max extremal n-pt correlators (ME) $k = n-2$
3. next to max extremal n-pt correlators (NME) $k = n-1$

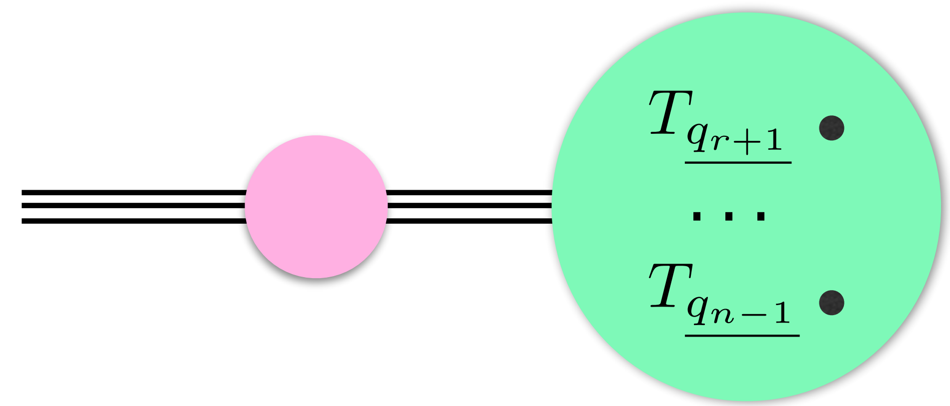
....

# PROOF of MULTI-POINT ORTHOGONALITY

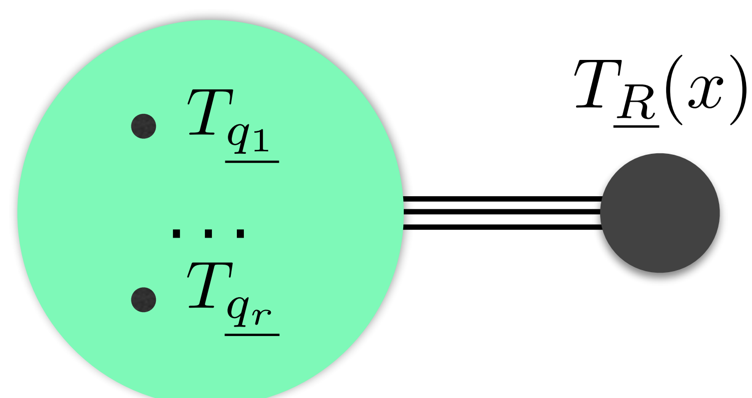



cut the starting point on the l.h.s.

$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R} \vdash R} C_{\underline{R}}\, T_{\underline{R}}(x) \;=$$

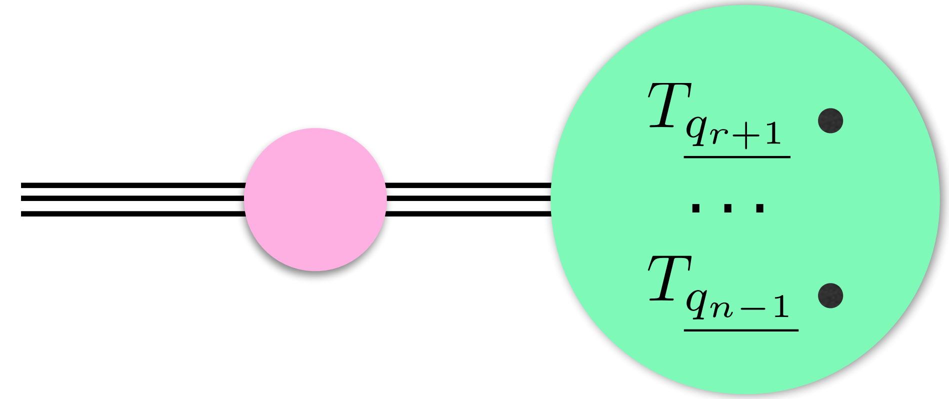



$$\mathcal{F}_{p|\underline{q_1}\dots\underline{q_{n-1}}} = \prod g_{ij}^{d_{ij}} \sum_{\underline{R} \vdash R} C_{\underline{R}}$$

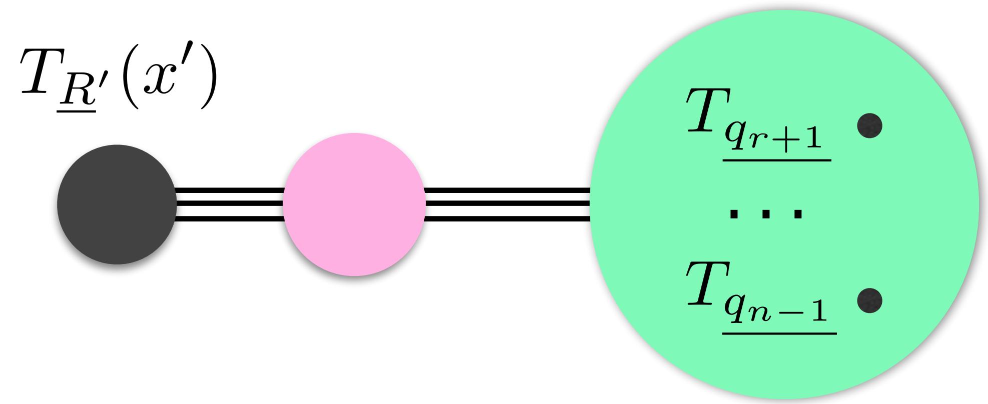



We need to find the $C_R$

$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R} \vdash R} C_{\underline{R}}\, T_{\underline{R}}(x) \; = $$

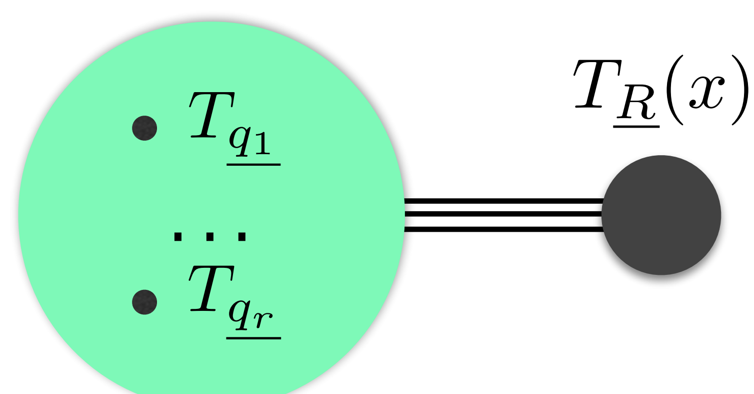



$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R'}, \underline{R} \vdash R} C_{\underline{R}}\, \langle T_{\underline{R'}}(x') T_{\underline{R}}(x) \rangle = $$

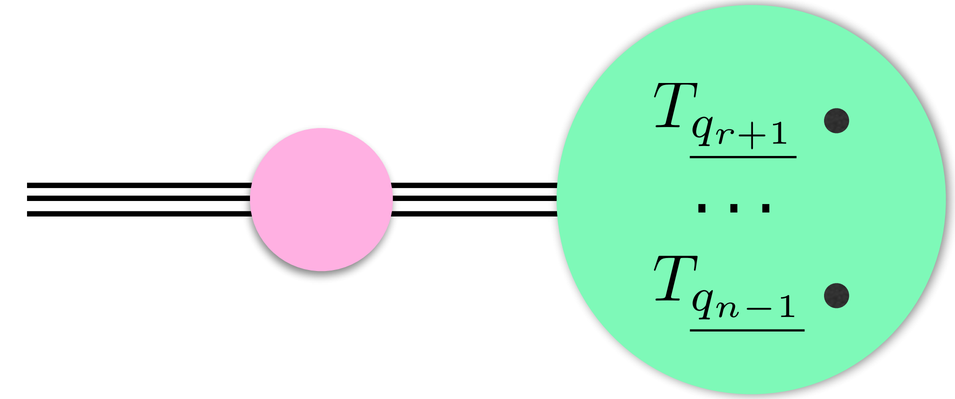



We find the $C_R$ by solving the above equation, then we substitute in

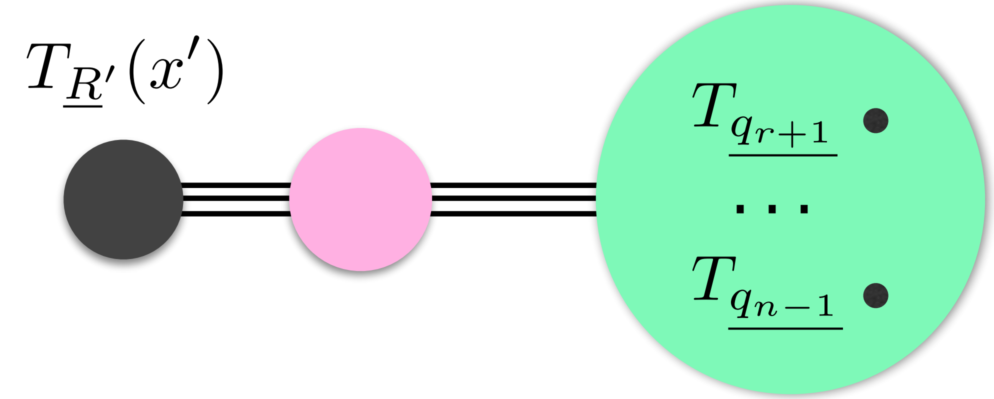
$$\mathcal{F}_{p|\underline{q_1} \ldots \underline{q_{n-1}}} = \prod g_{ij}^{d_{ij}} \sum_{\underline{R} \vdash R} C_{\underline{R}}$$



$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R} \vdash R} C_{\underline{R}}\, T_{\underline{R}}(x) \;=$$

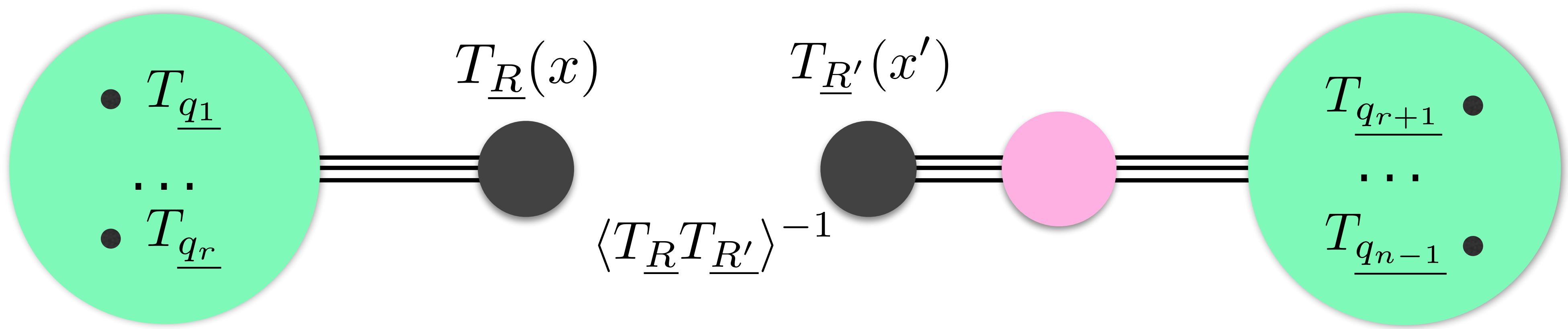



$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R'}, \underline{R} \vdash R} C_{\underline{R}}\, \langle T_{\underline{R'}}(x') T_{\underline{R}}(x) \rangle =$$

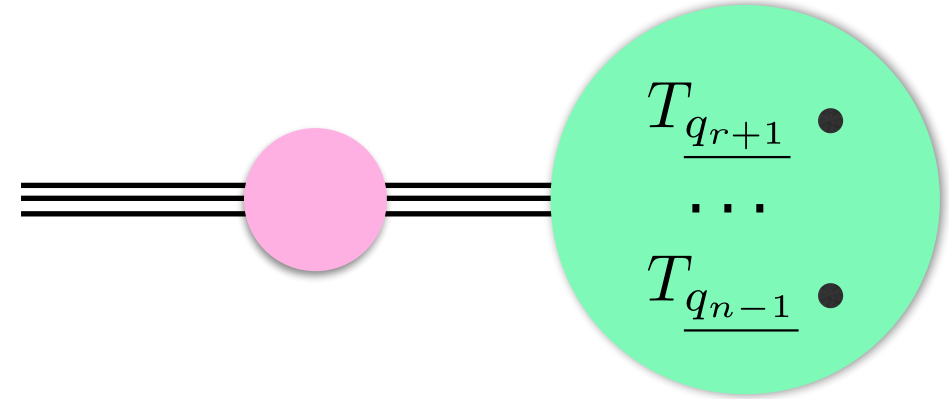



We find the $C_R$ by solving the above equation, then we substitute in

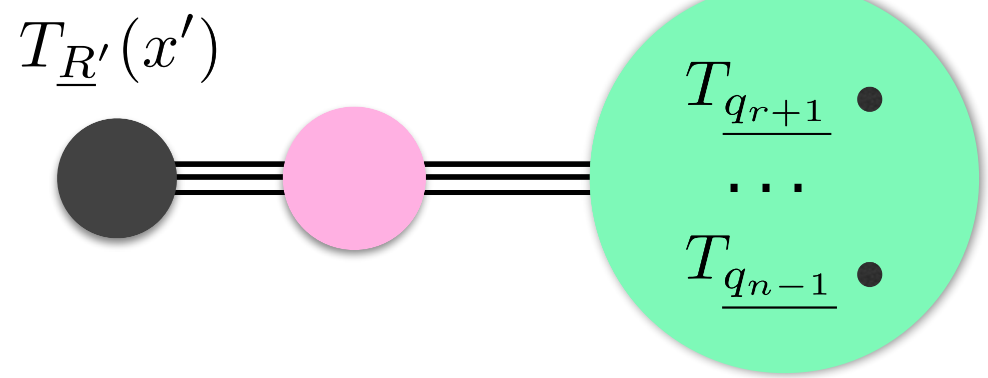
$$\mathcal{F}_{p|\underline{q_1}\ldots\underline{q_{n-1}}} \simeq$$



$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R} \vdash R} C_{\underline{R}}\, T_{\underline{R}}(x) \; =$$

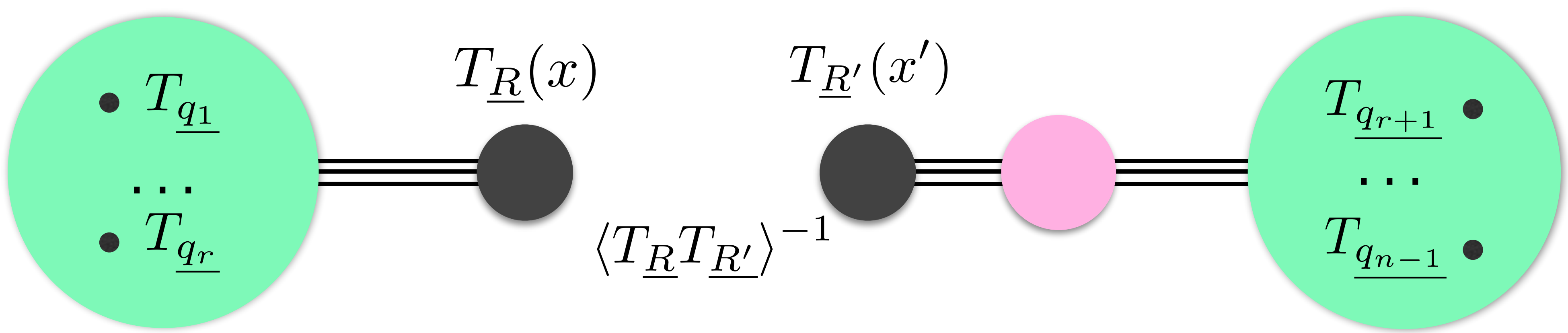



$$\prod g_{ij}^{d_{ij}} \sum_{\underline{R'},\underline{R} \vdash R} C_{\underline{R}}\, \langle T_{\underline{R'}}(x') T_{\underline{R}}(x) \rangle =$$



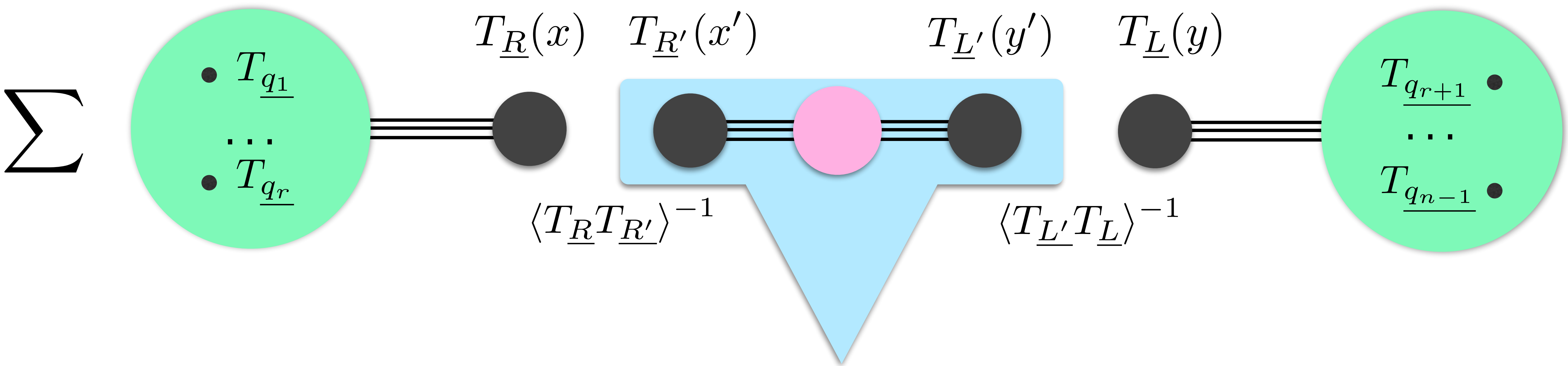

We find the $C_R$ by solving the above equation, then we substitute in

$$\mathcal{F}_{p|\underline{q_1}\ldots\underline{q_{n-1}}} \simeq$$




We will now repeat the same reasoning for the other green blob

$$\mathcal{F}_{p|\underline{q_1}\cdots\underline{q_{n-1}}} \simeq$$




This is now an extremal three-point function

$$= 0$$

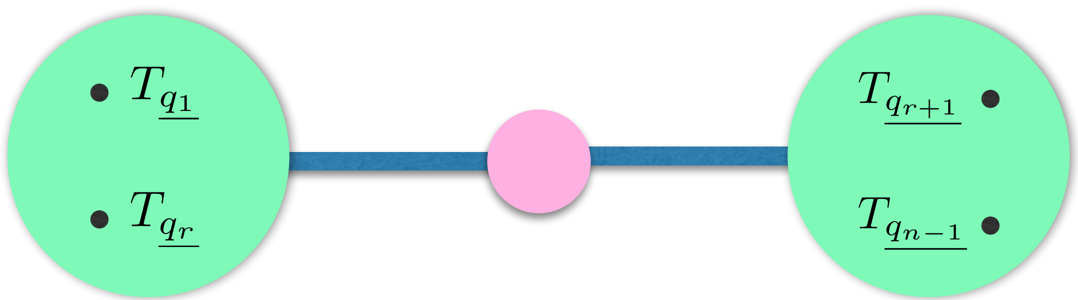
# APPLICATIONS

# SP operators look more complicated than Traces

and it would seem that correlators are even more complicated

## BUT

when we classifying correlators,
progressively away  from “dumbell” topologies




$$\frac{1}{2}\left(-p+\sum_{i=1}^{n-1} q_i\right)=k\geq 0$$

## WE FIND

1. near extremal n-pt correlators (NE)  $k\leq n-3$  exactly vanish !

The proof goes by showing that the only possible propagator structure is a dumbell, i.e. there are too many points and too less charges

*E. D’Hoker, J. Erdmenger, D. Z. Freedman and M. Perez-Victoria, [hep-th/0003218]

# 2. max extremal n-pt correlators (ME) $\quad k = n-2$

$$\langle \mathcal{O}_p(x)\mathcal{O}_{q_1}(x_1)\cdots\mathcal{O}_{q_{n-1}}(x_{n-1})\rangle_{\text{connected}} = \langle \mathcal{O}_p\mathcal{O}_p\rangle \left(\sum_{trees\,\mathcal{T}} |\mathcal{W}[\mathcal{T}]|\; \mathcal{T}\begin{bmatrix} d_1 & & b_{ij} \\ & \ddots & \\ & & d_{n-1}\end{bmatrix}\right)$$

$$|\mathcal{W}[\mathcal{T}]| = \prod_{i=1}^{n-1} q_i(q_i-1)\dots(q_i-d_i+1)$$

for 3-points $\quad p = q_1 + q_2 - 2$

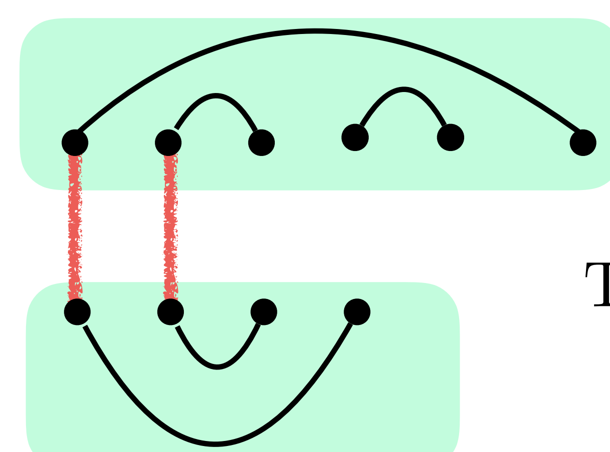
$$\langle \mathcal{O}_p(x)\mathcal{O}_{q_1}(x_1)\mathcal{O}_{q_2}(x_2)\rangle = \langle \mathcal{O}_p(x)T_{q_1}(x_1)T_{q_2}(x_2)\rangle \simeq q_1q_2\langle \mathcal{O}_p(x)T_p(y)\rangle = q_1q_2\langle \mathcal{O}_p(x)\mathcal{O}_p(y)\rangle$$

replacing SP with T
gives NE correlators = 0
but for the leading trace

only one Wick contraction

in double line notation, e.g.

or from OPE considerations



$$\mathrm{Tr}(\underbrace{\phi(y)\dots\dots\phi(y)}_{p})$$

## 2. max extremal n-pt correlators (ME) $\quad k = n - 2$

$$\langle \mathcal{O}_p(x)\mathcal{O}_{q_1}(x_1)\cdots\mathcal{O}_{q_{n-1}}(x_{n-1})\rangle_{\text{connected}} = \langle \mathcal{O}_p\mathcal{O}_p\rangle \left(\sum_{trees\,\mathcal{T}} |\mathcal{W}[\mathcal{T}]|\;\mathcal{T}\begin{bmatrix} d_1 & & b_{ij} \\ & \ddots & \\ & & d_{n-1}\end{bmatrix}\right)$$

The limit to two D3 branes and a SP

$$p - q = Q - 2$$

$$p, q, N \to \infty$$

$$p/N, q/N, Q \;\; \textit{fixed}$$

$$\langle \mathcal{O}_p(x)\mathcal{O}_q(x_1)\mathcal{O}_Q(x_2)\rangle = qQ\langle \mathcal{O}_p(x)\mathcal{O}_p(y)\rangle$$

from the Gamma function normalisations

$$\frac{\langle \mathcal{O}_p\mathcal{O}_q\mathcal{O}_Q\rangle}{\sqrt{\langle \mathcal{O}_p\mathcal{O}_p\rangle\langle \mathcal{O}_q\mathcal{O}_q\rangle\langle \mathcal{O}_Q\mathcal{O}_Q\rangle}} = \frac{qQ}{\sqrt{\langle \mathcal{O}_Q\mathcal{O}_Q\rangle}}\sqrt{\frac{\langle \mathcal{O}_p\mathcal{O}_p\rangle}{\langle \mathcal{O}_q\mathcal{O}_q\rangle}}$$

$$\to QN^Q$$

$$\to \frac{p^2}{q^2}e^{N(q-p)}N^{N(p-q)}e^{N(p-1)\log[1-p]-N(q-1)\log[1-q]}$$

$$\to \sqrt{Q}\,\frac{p}{N}\left(1-\frac{p}{N}\right)^{\frac{Q-2}{2}}$$

# 2. max extremal n-pt correlators (ME) $k = n-2$

$$\langle \mathcal{O}_p(x)\mathcal{O}_{q_1}(x_1)\cdots\mathcal{O}_{q_{n-1}}(x_{n-1})\rangle_{\text{connected}} = \langle \mathcal{O}_p\mathcal{O}_p\rangle \left(\sum_{trees\,\mathcal{T}} |\mathcal{W}[\mathcal{T}]|\; \mathcal{T}\begin{bmatrix} d_1 & & b_{ij} \\ & \ddots & \\ & & d_{n-1}\end{bmatrix}\right)$$

$$|\mathcal{W}[\mathcal{T}]| = \prod_{i=1}^{n-1} q_i(q_i-1)\ldots(q_i-d_i+1)$$

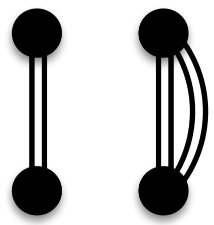
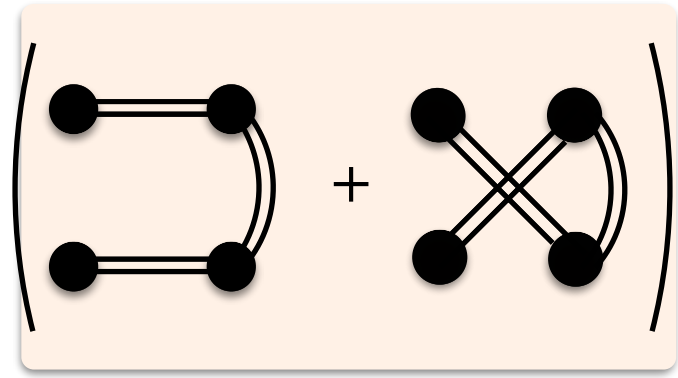
for n-points

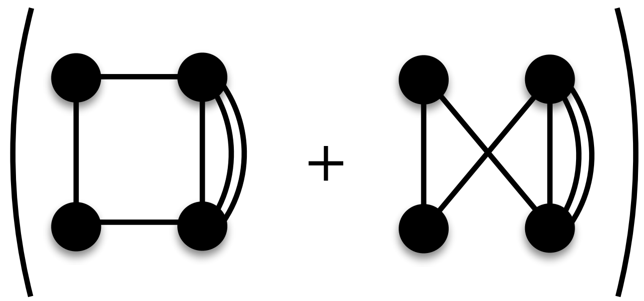
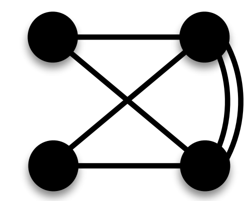
upon removing the SP there will be n-2 prop.
among n-1 points: this sub-diagram is a **tree**

# an example @ 4pt

$$\langle T_4(y) T_4(x_1) T_2(x_2) T_2(x_3) \rangle$$

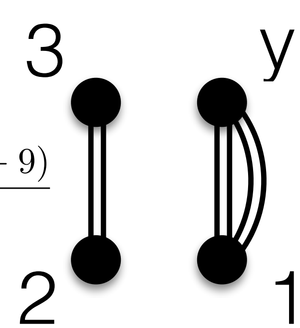
dumbbell ->

$\frac{8(N^2-1)^2(N^4-6N^2+18)}{N^2}$  $+ \frac{32(N^2-1)(2N^2-3)^2}{N^2}$  $+$

$\frac{64(N^2-1)(N^4-6N^2+18)}{N^2}$  $+ \frac{192(N^2-1)(N^4-6N^2+18)}{N^2}$ 

$$\langle \mathcal{O}_4(y) \mathcal{O}_4(x_1) \mathcal{O}_2(x_2) \mathcal{O}_2(x_3) \rangle$$

$$\langle \mathcal{O}_4 \mathcal{O}_4 \rangle = \frac{4(N^2-1)(N^2-4)(N^2-9)}{N^2+1}$$

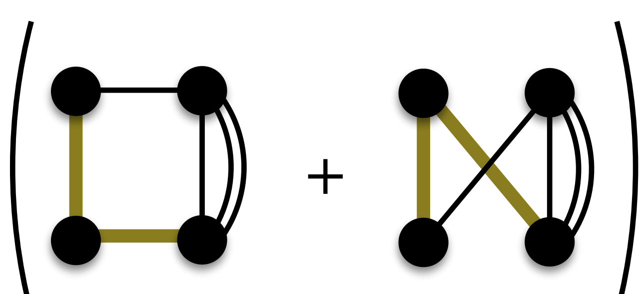
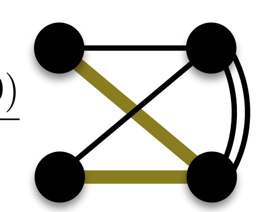
$\frac{8(N^2-1)^2(N^2-4)(N^2-9)}{N^2+1}$ 

$+$

**trees = 3pts and 2**

$q_1 q_2 q_3 (q_2 - 1) = 16$

$q_1 q_2 q_3 (q_1 - 1) = 48$

$\frac{64(N^2-1)(N^2-4)(N^2-9)}{N^2+1}$  $+ \frac{192(N^2-1)(N^2-4)(N^2-9)}{N^2+1}$ 

# MORE APPLICATIONS

# 3. next to max extremal n-pt correlators (NME) $k = n - 1$

for 3-points $p = q + Q - 4$

computable !

$$\langle \mathcal{O}_p(x) \mathcal{O}_q(x_1) \mathcal{O}_Q(x_2) \rangle = \langle \mathcal{O}_p(x) T_q(x_1) T_Q(x_2) \rangle + $$

$$+ \sum_{p_1=2}^{\frac{q_1}{2}} C_{p_1(q_1-p_1)} \langle \mathcal{O}_p(x) \left[ \mathcal{O}_{p_1} \mathcal{O}_{q-p_1} \right](x_1) \mathcal{O}_Q(x_2) \rangle \quad + \quad \textit{the other}$$

this is now a 4pt NME: use previous results! it follows that

$$\langle \mathcal{O}_p(x) \mathcal{O}_q(x_1) \mathcal{O}_Q(x_2) \rangle = \langle \mathcal{O}_p \mathcal{O}_p \rangle \Bigg[ qQ \left( N - \frac{(q-1)(Q-1)}{N} \right) \quad + $$

$$+ \sum_{p_1=2}^{\lfloor \frac{q_1}{2} \rfloor} C_{p_1(q-p_1)} Q(Q-1) p_1 (q-p_1) \quad + \quad \textit{the other} \Bigg]$$

everything is known in this formula

## 3. next to max extremal n-pt correlators (NME) $k = n - 1$

for 3-points $p = q + Q - 4$

the limit to two D3 branes and a SP

is found to be

$p, q, N \to \infty$

$p/N, q/N, Q$ *fixed*

$$\frac{\langle \mathcal{O}_p \mathcal{O}_q \mathcal{O}_Q \rangle}{\sqrt{\langle \mathcal{O}_p \mathcal{O}_p \rangle \langle \mathcal{O}_q \mathcal{O}_q \rangle \langle \mathcal{O}_Q \mathcal{O}_Q \rangle}} \to \sqrt{Q}\,\frac{p}{N}\left(1 - \frac{p}{N}\right)^{\frac{Q-4}{2}}\left(1 - \frac{(Q-1)p}{2N}\right)$$

an independent computation of 2103.16580 matches this formula

[Yang, Jiang, Komatsu,Wu]

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for n-points: similar manipulations as for the case of 3-points yield a sum of ME correlators holds

# 4. what about NNME or beyond NME

things obviously get complicated BUT …

a first look at 3-points, e.g.

$$\langle T_6 T_6 T_6 \rangle = \frac{72(N^2-1)(456000+96000N^2-134400N^4+34200N^6-4388N^8+295N^{10}+3N^{12}}{N^6}$$

$$\langle T_6 T_6 T_{[4,2]} \rangle = \frac{216(N^2-1)(N^2-4)(30400-1600N^2-1040N^4-15N^6+21N^8)}{N^5}$$

$$\langle T_6 T_6 T_{[3,3]} \rangle = \frac{576(N^2-1)(-34200+8400N^2+2220N^4-710N^6+51N^8+6N^{10})}{N^5}$$

$$\langle T_6 T_6 T_{[2,2,2]} \rangle = \frac{1152(N^2-1)(5700-2700N^2+630N^4-126N^6+29N^8)}{N^4}$$

*Ancillary file in [2007.09395]*

NOT GUESSABLE

final result much simpler than intermediate steps

$$\langle \mathcal{O}_6 T_6 T_6 \rangle = \left(300 + \frac{7200}{N^2} + 36N^2\right) \langle \mathcal{O}_6 \mathcal{O}_6 \rangle$$

! GUESSABLE !

Other cases: final resul similarly simple

vs more and more complicated intermediate combinatorics

# SUMMARY & CONCLUSIONS

We have explicit ways of writing the SP operators

We know the 2pt function normalisation

We know that SP interpolate between T and giant gravitons

We have a multi-point orthogonality theorem

instances of this were shown to be important for the consistency of the AdS5xS5 truncation to the graviton multiplet

**E. D'Hoker, J. Erdmenger, D. Z. Freedman and M. Perez-Victoria, [hep-th/0003218]*

We understand ME, NME correlators, well

SPECULATION?

after all the SP are dual to the Traces!
SP might enjoy integrable structure too !!

THANKS

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