Chiral Composite Linear Dilaton:

2d YM, Symmetric Product Orbifold, and Generalized Veneziano

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Based on 2506.21663, 2511.16280, and 2512.xxxxx with
Pronobesh Maity (EPFL)

Chiral Composite Linear Dilaton

- We propose a novel string worldsheet theory "Chiral Composite Linear Dilaton"

$$S_{\text{CLD}} = \int \frac{d^2z}{2\pi}\left[\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + q\left(\partial\varphi\bar{\partial}\varphi + \frac{\hat{R}}{2}\varphi\right)\right]$$

  - β : (1,0) form, Lagrange multiplier.
  - γ : (0,0) form, a complex target-space coordinate.
  - $\varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma})$. A composite linear dilaton.
  - Highly non-linear action, but computable.
  - Integrating out $\beta \Rightarrow \bar{\partial}\gamma = 0$: localization to holomorphic maps.
  - Central charge $c_{\text{CLD}} = 2 + 24q$.

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  - Central charge $c_{\text{CLD}} = 2 + 24q$.

- Worldsheet descriptions of
  - Chiral 2d YM.
  - Symmetric product orbifold of arbitrary seed CFT with $c < 24$.
  - Generalized Veneziano amplitude.

Plan

1. Introduction and Motivation

2. Action, Stress tensor and OPE

3. Torus partition function

4. Amplitudes

5. Conclusion

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Motivation 1: gauge/string duality

- AdS/CFT and gauge/string duality have been and still are a rich source of inspiration and progress.

  Strongly coupled phenomena in QFT ↔ Semiclassical gravity / string

  Concrete computation in QFT ↔ Conceptual questions / conjectures in QG

- But for further progress & generalization, it is important to understand the inner working of known correspondences.

  String dual of confining gauge theory?

  Holography for cosmological spacetime / closed universe?

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- One strategy: Focus on cases in which QFT is simple.
  - Symmetric product orbifold of T^4 $\leftrightarrow$ String on $AdS_3 \times S^3$ with $k = 1$

    Eberhardt, Gabrediel, Gopakumar
  - Free $\mathcal{N} = 4$ SYM $\leftrightarrow$ String in (ambi-)twistor space?

    Gopakumar, Witten, Berkovits, Gaberdiel, Gopakumar
  - Twisted holography

    Costello, Gaiotto, Paquette, ….

- Still, $\exists$ many other simple large N QFTs with no known duals.
  - Symmetric product orbifold of other seed CFTs?
  - 2d YM (the simplest confining gauge theory)

Motivation 2: generalized Veneziano

- Dual resonance amplitudes: precursor of string theory

$$A(s,t) = \frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-s-t)}$$

Veneziano amplitude

$$A(s,t,u) = \frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$

Virasoro-Shapiro amplitude

- Remarkable properties
  - Linear Regge trajectories, infinite resonances.
  - Resonances in different channels in a single mathematical expression.
  - Unitary below $d \leq 10$.
  - Consistent weakly interacting amplitudes of higher spin particles.

- Revival of interest
  - **Uniqueness**: How unique are they?
  - **Search for large N QCD amplitudes**: Are there deformations that describe meson amplitudes of large N QCD?
  - **S-matrix bootstrap**

Cheng, Remmen, Arkani-Hamed, Figueiredo, Sciotti, Tarquini…

Albert, Knopp, Henriksson, Rastelli, Vichi, Jepsen, Haring, Zhiboedov, Berman, Elvang, Geiser, Lin, Eckner, Figueroa, Tourkine,…

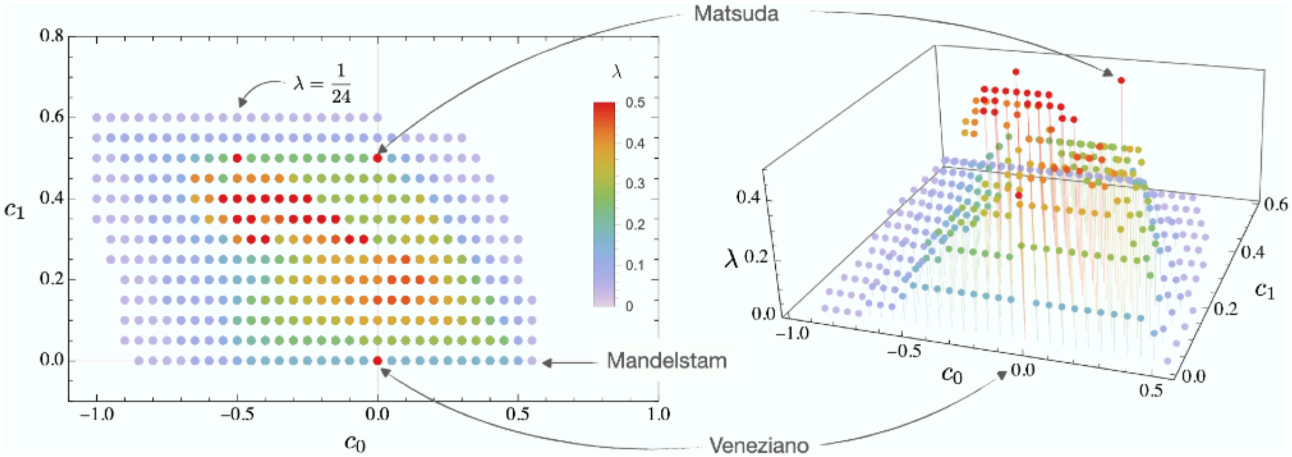
- ∃ many generalizations of 4-pt amplitudes.

Motivation 2: generalized Veneziano

- A class of generalized Veneziano amplitudes Mandelstam

$$A(s,t) = \frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-s-t)}\, {}_3F_2\left(-s,-t,-\delta;-\frac{s+t}{2},\frac{1-s-t}{2};\lambda\right)$$

- Unitarity in $D=4$ was recently analyzed in Haring, Zhiboedov



- No worldsheet description, no higher-point generalization known.

CLD as a common answer

- $\exists$ many other simple large N QFTs with no known duals.
  - Symmetric product orbifold of other seed CFTs?
  - 2d YM (the simplest confining gauge theory)

- Worldsheet description of generalized Veneziano amplitudes?

CLD as a common answer

- $\exists$ many other simple large N QFTs with no known duals.
  - Symmetric product orbifold of other seed CFTs?
  - 2d YM (the simplest confining gauge theory)

- Worldsheet description of generalized Veneziano amplitudes?

- Chiral Composite Linear Dilaton is an answer to both questions.

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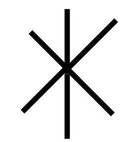
Action and stress tensor

$$S_{\rm CLD} = \int \frac{d^2 z}{2\pi} \left[\beta \bar\partial \gamma + \bar\beta \partial \bar\gamma + q \left(\partial\varphi \bar\partial \varphi + \frac{\hat R}{2}\varphi \right) \right]$$

$$\varphi = \log(\partial\gamma\bar\partial\bar\gamma)$$

- Linear in β . Highly non-linear in γ.



- $\gamma\gamma$ and $\beta\gamma$ OPE is unaffected by nonlinearity:

$$\gamma(z)\gamma(w) \sim \text{regular} \qquad \beta(z)\gamma(w) \sim -\frac{1}{z-w}$$

- $\beta\beta$ OPE is non-linear and non-standard:

$$\beta(z)\beta(w) \sim 2q\partial_z\partial_w \left[\frac{1}{(z-w)^2} \frac{1}{\partial_z\gamma(z)\partial_w\gamma(w)} \right]$$

Derived from path integral (later)

- Stress tensor can be derived from $\delta/\delta g_{ab}$:

$$T(z) = -\beta\partial\gamma + 2q\{\gamma, z\}$$

$$\{f,z\} = \frac{\partial^3 f}{\partial f} - \frac{3}{2}\left(\frac{\partial^2 f}{\partial f}\right)^2$$

Stress tensor OPE

$$T(z) = -\beta\partial\gamma + 2q\{\gamma, z\}$$

$$\gamma(z)\gamma(w) \sim \text{regular} \qquad \beta(z)\gamma(w) \sim -\frac{1}{z-w} \quad \beta(z)\beta(w) \sim 2q\partial_z\partial_w\left[\frac{1}{(z-w)^2}\frac{1}{\partial_z\gamma(z)\partial_w\gamma(w)}\right]$$

- Using OPEs, stress tensor OPEs can be computed (simplification: stress tensor is linear in β)

$$T(z)T(w) \sim \frac{1+12q}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{z-w}$$

Consistent with conformal symmetry. Central charge $c = 2 + 24q$

- To derive it, one sometimes needs to deal with

$$\beta(z)\log\partial\gamma(w) \sim -\frac{1}{(z-w)^2}\frac{1}{\partial\gamma(w)}$$

Can be derived from "replica trick"

$$\beta(z)(\partial\gamma(w))^n = -\frac{n}{(z-w)^2}(\partial\gamma)^{n-1}$$

- It is also easy to check that γ is $(0{,}0)$ and β is $(1{,}0)$ operators.

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Dual to Symmetric Product Orbifold

- We propose that string dual to symmetric product orbifold of seed CFT $S_{\rm seed}$ is given by

$$S = S_{\rm CLD} + S_{\rm seed}$$

$$S_{\rm CLD} = \int \frac{d^2 z}{2\pi} \left[\beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma} + q \left(\partial \varphi \bar{\partial} \varphi + \frac{\hat{R}}{2} \varphi \right) \right]$$

$$q = 1 - \frac{c_{\rm seed}}{24}$$

- Evidence: torus partition function (torus target space)

$$\gamma \sim \gamma + 2\pi R \sim \gamma + 2\pi R \zeta$$

- Integrating out β gives localization to $\bar{\partial}\gamma = 0$ (holomorphic map).
- In addition, any maps with "branch point" $\partial\gamma(z^*) = 0$ is suppressed since $\partial\gamma(z^*) = 0 \quad \rightarrow \quad \varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma}) = -\infty \quad \rightarrow \quad e^{-S_{\rm CLD}} = 0\,.$
- Such maps exist only when the worldsheet is also torus (genus 1)

$$\frac{\gamma}{R} = \frac{(M - \bar{\tau} W) z + (\tau W - M) \bar{z}}{\tau - \bar{\tau}}$$

$$M = m^1 + \zeta m^2 \qquad W = w^1 + \zeta w^2$$

$$\tau = \frac{M}{W}$$

Details on partition function

$$\frac{\gamma}{R} = \frac{(M - \bar{\tau} W) z + (\tau W - M)\bar{z}}{\tau - \bar{\tau}}$$

$$M = m^1 + \zeta m^2$$
$$W = w^1 + \zeta w^2$$

$$\tau = \frac{M}{W}$$

- More concretely, the path integral gives

$$\sum_{\{m^a, w^a\}} \frac{\zeta_2}{|W|^2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \delta^{(2)}\left(\tau - \frac{M}{W}\right) Z_{\text{seed}}(\tau, \bar{\tau})$$

- Localization of the worldsheet moduli & a sum over four integers.

- Extend the integration domain to the upper half plane using two of the four integers and evaluate the delta function:

$$\sum_{a,d=1}^{\infty} \frac{p^{ad}}{ad} \sum_{b=0}^{d-1} Z_{\text{seed}}\left(\frac{b + a\zeta}{d}, \frac{b + a\bar{\zeta}}{d}\right)$$

$$p = e^{B\text{Area}/\pi}$$

$$\delta S = -B \int \frac{d^2 z}{4\pi} \left(\partial\gamma\bar{\partial}\bar{\gamma} - \bar{\partial}\gamma\partial\bar{\gamma}\right)$$

- Coincides with the grand canonical free energy of symmetric product orbifold (Hecke operator expression)

Lightening review of chiral 2d YM

- 2d YM: Simplest possible confining theory.

  Linear potential, area law, meson spectrum…..

- Completely solvable even at finite N:

$$Z_{\mathscr{M}} = \sum_{\text{rep of U(N)}} \left(\dim R\right)^{2-2G} e^{-g_{\text{YM}}^2 A C_2(R)}$$

- At large N, the result factorizes into "chiral" and "anti-chiral" parts

$$Z_{\mathscr{M}} \overset{\text{large N}}{\rightarrow} Z_{\mathscr{M}}^{\text{chiral}} Z_{\mathscr{M}}^{\text{anti}}$$

- At finite N, each partition function receives corrections. In addition there will be interactions between chiral and anti-chiral parts.

$$Z_{\mathscr{M}}^{\text{chiral}} = Z_0^{\text{chiral}} + \frac{Z_1^{\text{chiral}}}{N^2} + \cdots \qquad Z_{\mathscr{M}} = Z_{\mathscr{M}}^{\text{chiral}} Z_{\mathscr{M}}^{\text{anti}} + \frac{Z_{\text{int}}}{N^2} + \cdots$$

2d YM as string theory

- Gross Taylor: 1/N expansion can be interpreted as string genus expansion.
- Chiral and anti-chiral parts correspond to worldsheets wrapping the target space with different orientations.
- But the explicit worldsheet action was not written down.

2d YM as string theory

- Gross Taylor: 1/N expansion can be interpreted as string genus expansion.
- Chiral and anti-chiral parts correspond to worldsheets wrapping the target space with different orientations.
- But the explicit worldsheet action was not written down.
- Horava: Proposed a "rigid topological string". Much more complicated than standard string theory.
- Cordes, Moore, Rangoolam: "zero area limit" of 2d YM can be interpreted as topological string.
- Vafa: topological string dual to 2d YM on a torus.
- Aganagic, Ooguri, Saulina, Vafa: Partition functions on Riemann surfaces coincide with q-deformed topological string partition function.

Our string dual

$$S = S_{\rm CLD}\big|_{q=1} + \int \frac{d^2 z}{4} (\partial\gamma\bar{\partial}\bar{\gamma} - \bar{\partial}\gamma\partial\bar{\gamma})$$

- Symmetric product orbifold of "nothing"
- Torus world sheet to torus target space path integral can be evaluated in a similar manner:

$$\sum_{K=1}^{\infty} \frac{e^{-K\lambda A/2}}{2K} \sum_{\substack{ad=K \\ a,d\in\mathbb{Z}_+}} (a+d)$$

- Coincides with the large N free energy of chiral 2d YM by Gross and Taylor.

A technical but important remark

- At this order (genus 1), the symmetric product orbifold and 2d YM are similar.
- But at subleading order of g_s, there is an important difference.
- The answer for the symmetric product orbifold is "genus-1 exact". No higher-genus correction. But the chiral 2d YM partition function is known to receive 1/N corrections.
- At first sight, this is a problem for our proposal on chiral 2d YM.

Any maps with "branch point" $\partial\gamma(z^*) = 0$ is suppressed since

$$\partial\gamma(z^*) = 0 \quad \to \quad \varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma}) = -\infty \quad \to \quad e^{-S_{\rm CLD}} = 0\,.$$

- Maps at higher order always contain "branch points" $\partial\gamma = 0$

$$?$$

Renormalization of g_s

$$\partial\gamma(z^*) = 0 \quad \rightarrow \quad \varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma}) = -\infty \quad \rightarrow \quad e^{-S_{\text{CLD}}} = 0\,.$$

- There is a famous resolution to this known in light cone string field theory.

  Mandelstam, Baba, Ishibashi, Murakami,…

- The key point is that # of "branch points" ($\partial\gamma = 0$) is equal to Euler characteristics (n+ 2g -2).

- Thus one can absorb the divergence of φ into a renormalization of g_s:

$$g_s \times \text{“}0\text{”} = g_s^{\text{ren}}$$

- In 2d YM (and generalized Veneziano), we always perform this renormalization while, for symmetric product orbifold, we do not.

Comment 1: Non-relativistic string

Gomis, Ooguri,,…, Bergshoff, Gomis, Yan,….

$$\int d^2z\,\left(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X}\right) + \text{(Polyakov action for transverse modes)}$$

- Worldsheet theory is relativistic, target space spectrum is non-relativistic.

- It can be obtained by a double-scaling limit of usual string theory in which one sends B-field large (critical) and $\alpha' \to 0$.
(Open string sector becomes noncommutative gauge theory)

- Roughly speaking, the non-relativistic limit kills half of the spectrum.

- Here taking the limit makes the theory chiral and kill "anti-strings" which do not exist in the symmetric product orbifolds.

Comment 2: Tensionless AdS3/CFT2

Eberhardt, Gaberdiel, Gopakumar

- Free field representation of SL(2,R) WZW:

$$\int d^2z \left(\beta\bar{\partial}X + \bar{\beta}\partial\bar{X} + 4\underline{\partial\Phi\bar{\partial}\Phi} - e^{-2\Phi}\beta\bar{\beta} - R\Phi\right)$$

  - Similar to our action, but Φ is not composite.

- They argued that, for tensionless string (k=1), the path integral localizes to

$$\Phi = -\log\epsilon + \log\partial X\bar{\partial}\bar{X}$$

- After the replacement, the action has the same structure as ours.

- The relation is not rigorous: their theory is supersymmetric, our theory is purely boson.

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Proposal

- Worldsheet action

$$S = S_{\mathbb{R}^{d-1,1}} + S_{\beta\gamma} + S_{\chi} + S_{bc}$$

Standard Polyakov action
in d dimensions

$$S_{\mathbb{R}^{d-1,1}} = \int_{\Sigma} \frac{d^2 z}{2\pi\alpha'} \partial X^{\mu} \bar{\partial} X_{\mu}$$

Proposal

- Worldsheet action

$$S = S_{\mathbb{R}^{d-1,1}} + S_{\beta\gamma} + S_{\chi} + \underline{S_{bc}}$$

bc-ghost

Proposal

- Worldsheet action

$$S = S_{\mathbb{R}^{d-1,1}} + S_{\beta\gamma} + S_{\chi} + S_{bc}$$

Arbitrary "internal" CFT with central charge c_χ

Proposal

- Worldsheet action

$$S = S_{\mathbb{R}^{d-1,1}} + \boxed{S_{\beta\gamma}} + S_{\chi} + S_{bc}$$

Chiral Composite Linear Dilaton

Proposal

- Worldsheet action

$$S = S_{\mathbb{R}^{d-1,1}} + S_{\beta\gamma} + S_{\chi} + S_{bc}$$

Chiral Composite Linear Dilaton

- "Open" Chiral Composite Linear Dilaton (CLD) SK, Maity 2506.21663

$$S_{\beta\gamma} = \int_{\Sigma} \frac{d^2 z}{2\pi} \left[\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + q \left(\partial\varphi\bar{\partial}\varphi + \frac{\hat{R}}{2}\varphi \right) \right] + \frac{q}{\pi} \int_{\partial\Sigma} ds\, \hat{k}\varphi$$

open string boundary term

$$\varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma})$$

Proposal

- Worldsheet action

$$S = S_{\mathbb{R}^{d-1,1}} + S_{\beta\gamma} + S_{\chi} + S_{bc}$$

Most important part
"Chiral Composite Linear Dilaton"

- "Open" Chiral Composite Linear Dilaton (CLD) SK, Maity 2506.21663

$$S_{\beta\gamma} = \int_\Sigma \frac{d^2 z}{2\pi} \left[\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + q \left(\partial\varphi\bar{\partial}\varphi + \frac{\hat{R}}{2}\varphi \right) \right] + \frac{q}{\pi} \int_{\partial\Sigma} ds\, \hat{k}\varphi$$

open string boundary term

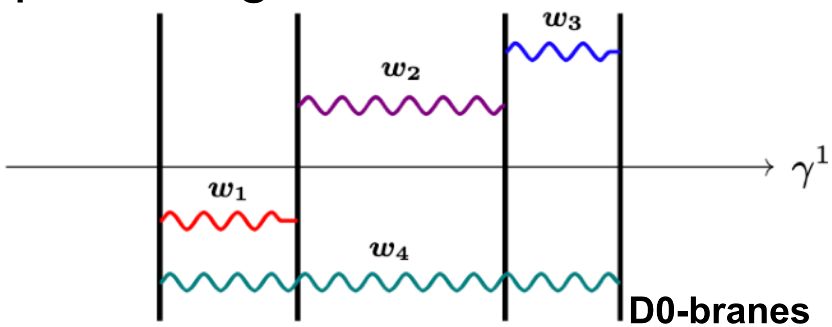
$$\varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma})$$

- $\hat{R}$ and $\hat{k}$ are worldsheet Ricci curvature and geodesic curvature.
- Central charge $c_{\beta\gamma} = 2 + 24q$. Conformal anomaly cancels when $q = 1 - \dfrac{d + c_\chi}{24}$

Set up

$$S_{\beta\gamma} = \int_\Sigma \frac{d^2z}{2\pi}\left[\beta\bar\partial\gamma + \bar\beta\partial\bar\gamma + q\left(\partial\varphi\bar\partial\varphi + \frac{\hat{R}}{2}\varphi\right)\right] + \frac{q}{\pi}\int_{\partial\Sigma} ds\,\hat{k}\varphi \qquad \varphi = \log(\partial\gamma\bar\partial\bar\gamma)$$

- Consider open strings stretched between "D0-branes" in γ-plane



- Boundary conditions

$$\delta\gamma^1 \propto \delta(\gamma + \bar\gamma) = 0 \qquad \partial_n\gamma^2 \propto \partial_n(\gamma - \bar\gamma) = 0 \qquad \beta + \bar\beta = 0$$

Dirichlet — Neumann — From EOM

Set up

$$S_{\beta\gamma} = \int_\Sigma \frac{d^2 z}{2\pi}\left[\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + q\left(\partial\varphi\bar{\partial}\varphi + \frac{\hat{R}}{2}\varphi\right)\right] + \frac{q}{\pi}\int_{\partial\Sigma} ds\,\hat{k}\varphi \qquad \varphi = \log(\partial\gamma\bar{\partial}\bar{\gamma})$$

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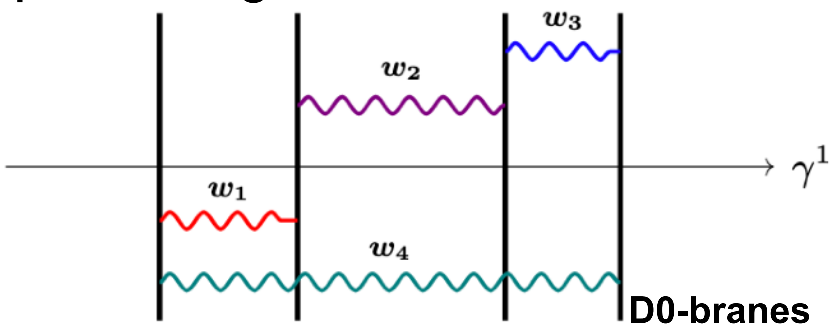


- Boundary conditions

$$\delta\gamma^1 \propto \delta(\gamma + \bar{\gamma}) = 0$$
Dirichlet

$$\partial_n\gamma^2 \propto \partial_n(\gamma - \bar{\gamma}) = 0$$
Neumann

$$\beta + \bar{\beta} = 0$$
From EOM

- Vertex operators

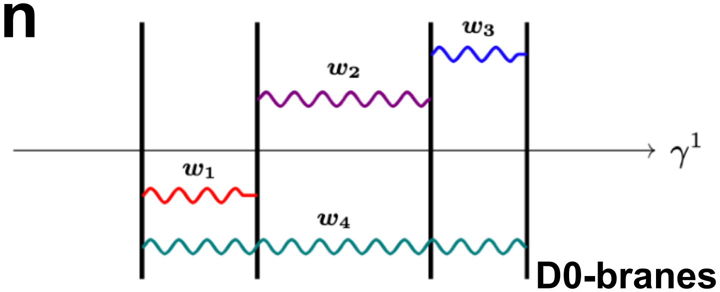
$$e^{ikX} e^{iwR\int \beta - \bar{\beta}}$$

$$(h, \bar{h}) = (\alpha' k^2 + q, \alpha' k^2 + q)$$

On-shell condition: $\alpha' k^2 = 1 - q$

- For general q, strings are not massless.

Computation



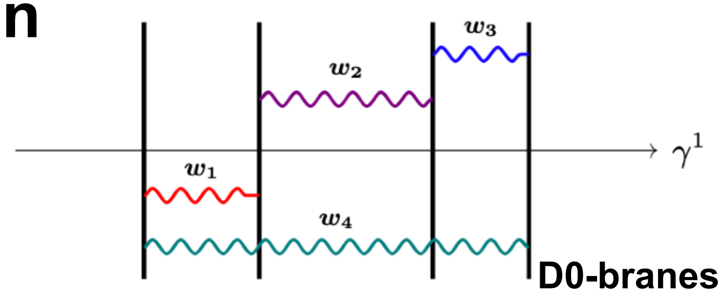
- Integrate out β's in the presence of vertex operators

$$S_{\beta\gamma} = \int_\Sigma \frac{d^2z}{2\pi}\left[\beta\bar\partial\gamma + \bar\beta\partial\bar\gamma + q\left(\partial\varphi\bar\partial\varphi + \frac{\hat R}{2}\varphi\right)\right] + \frac{q}{\pi}\int_{\partial\Sigma} ds\,\hat k\varphi \qquad e^{ikX}e^{iwR\int\beta-\bar\beta}$$

- Worldsheet localizes to "Mandelstam map"

$$\gamma(z) = \rho(z) = -i\sum_k w_k R\log(z - z_k)$$

Computation



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- Worldsheet localizes to "Mandelstam map"

$$\gamma(z) = \rho(z) = -i \sum_k w_k R \log(z - z_k)$$

- Evaluate the CLD action on Mandelstam map

Mandelstam, Baba, Ishibashi, Murakami

$$e^{-S_{\mathrm{CLD}}[\rho]} = \frac{|w_1 w_2 w_3 w_4|^{q/2} |w_1 + w_3|^q}{|z_{12} z_{14} z_{23} z_{24}|^q} \cdot |z_c - a_+|^{q/2} |z_c - a_-|^{q/2}$$

- $z_c = \dfrac{z_{12} z_{34}}{z_{13} z_{24}}$ is a cross ratio.
- $a_\pm = \dfrac{1}{2} \pm \dfrac{i}{2} \sqrt{\dfrac{w_2}{w_1} \left(2 + \dfrac{w_2}{w_1} \right)}$ are related to "interaction points" (their discriminant): $\partial \rho = 0$

Computation 2

- Combining it with Koba-Nielsen factor from S_X gives

$$\int_0^1 dx\, x^{-\alpha' s+q-2}\,(1-x)^{-\alpha' t+q-2}\,\left[(x-a_+)(x-a_-)\right]^{q/2}$$

- Basically of the same form as the Mandelstam amplitude:

$$A(s,t) = \frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-s-t)}\,{}_3F_2\left(-s,-t,-\delta;-\frac{s+t}{2},\frac{1-s-t}{2};\lambda\right)$$

$$A(s,t) = \int_0^1 dz\, z^{-s-b-1}(1-z)^{-t-b-1}\,(1-4\lambda z(1-z))^{\delta}$$

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$$A(s,t) = \int_0^1 dz\, z^{-s-b-1}(1-z)^{-t-b-1}\,(1-4\lambda z(1-z))^{\delta}$$

- $a_\pm$ can be tuned by changing the ratio w_2/w_1. ($a_\pm = \frac{1}{2} \pm \frac{i}{2}\sqrt{\frac{w_2}{w_1}\left(2+\frac{w_2}{w_1}\right)}$)

  (We already fixed $w_{3,4}$ so that the amplitude becomes s-t symmetric $a_+ = 1 - a_-$)

- q can also be modified by changing the central charge of internal CFT S_χ.
- An unsatisfactory feature: b is constrained and external states are not massless.

  → Can be remedied by including free boson in the internal CFT and dressing vertex operators by free boson vertex operators.

Closed string generalization

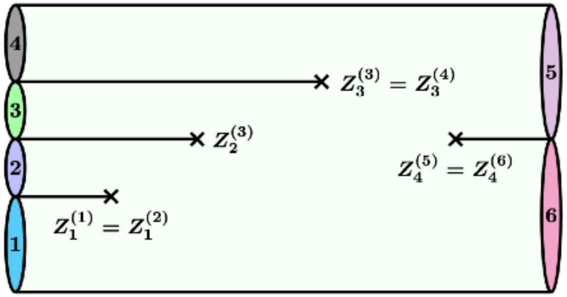
- Performing a similar computation in closed string leads

$$\mathcal{A}_4^{\text{closed}} = \int_{\mathbb{C}} d^2 z \, |z|^{-\frac{\alpha'}{2}s-2b-2} |1-z|^{-\frac{\alpha'}{2}t-2b-2} \underline{|1-4\lambda z(1-z)|^q}$$

2 zeros

- Generalization of KLT trick gives a sum of products of Appel $\mathbb{F}_1$

- It is only s-t symmetric (not s-t-u symmetric)
  - Technical reason: One cannot make it s-t-u symmetric using just 2 zeros.
  - Conceptual reason: Conservation of $w's$ $\sum w_i = 0$. Some $w's$ are positive and others are negative.Mandelstam map is a light cone diagram.



- Similarly, open-string higher-point amplitudes are only partially crossing symmetric.

Summary

$$S_{\mathrm{CLD}} = \int \frac{d^2 z}{2\pi} \left[\beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma} + q \left(\partial \varphi \bar{\partial} \varphi + \frac{\hat{R}}{2} \varphi \right) \right]$$

- Proposed a novel worldsheet theory that is potentially dual to chiral 2d YM, symmetric product orbifold and generalized Veneziano amplitude.
- Non-linear action, OPE contains inverse fields. Nevertheless the path integral can be performed explicitly.

- Understand the connection to topological string for chiral 2d YM on a torus by Vafa

  Speculation: topological string contains fermionic modes on the world sheet.
  Integrating them out would lead to CLD?

  If so, we will have a better formulation also for dual of symmetric product orbifold and world sheet description of generalized Veneziano.

- More evidence for the duality with symmetric product orbifold.
- Other generalized Veneziano amplitudes?
- Crossing symmetric generalized Virasoro-Shapiro?
- “Derivation” of duality for symmetric product orbifold. Lunin, Mathur